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TECHNICAL NOTE

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STATISTICAL WIND DISTRIBUTION DATA FOR USE AT NASA WALLOPS STATION

By William L. Weaver, Andrew G. Swanson, and John F. Spurling

Langley Research Center
Langley Station, Hampton, Va.

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SUMMARY

Wind data from radiosonde measurements made at Norfolk, Virginia, and Washington, D.C., were utilized to determine statistical wind distribution data for the NASA Wallops Station, Wallops Island, Virginia, for an altitude range from 4,850 to 86,150 feet. The statistical analysis was based on the bivariate normal distribution method. Some of the results obtained by the binormal analysis are compared with results obtained empirically. The assumption of a circular distribution is shown to be valid for winds at Wallops Island. A description of the method of interpolating the original data from Washington and Norfolk and an outline of the statistical methods employed are included. Simple methods are discussed for obtaining circular distribution wind speed, wind direction, and component wind probabilities and also elliptical distribution component probabilities.

INTRODUCTION

Statistical wind data for the NASA Wallops Station are needed both for rocket vehicle design and for flight test operations at that site. These wind data are required in analyses of both guided and unguided vehicles, particularly as structural design and dispersion criteria become more stringent with the advent of high-performance rocket systems.

In the past most wind studies have been restricted to empirical methods (particularly the cumulative percentage frequency method). Results from studies based on these empirical methods are usually limited in both scope and accuracy. Accuracy in most cases is highly limited in the regions of extreme wind speeds. A statistical approach to the estimation of wind distributions is therefore desirable which is not adversely affected by minor bias or extreme values in the data. The method employed herein is believed to satisfy these requirements. It treats the wind vector components as normally distributed random variables.

Crutcher (ref. 1) suggests this bivariate normal distribution method and discusses briefly its implementation. Reference 2 also gives a brief discussion of this method. In reference 3, Crutcher makes use of the bivariate normal method to determine parameters describing wind distributions in the Northern Hemisphere by utilizing data from several stations. Reference 4 provides tables for a closely related problem involving the bivariate normal distribution.

All these publications will aid in the understanding of the bivariate treatment of wind distributions but, because this report is intended primarily for engineers rather than for meteorologists and statisticians, an appendix is included which further outlines this particular statistical approach to the study of winds.

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Any statistical analysis is dependent upon having available a sufficiently large sampling of observations and, because no large samplings were available for the NASA Wallops Station, wind sampling data from Washington, D.C., and Norfolk, Va., were interpolated to arrive at wind distributions for Wallops Island. Because of the proximity of these two stations, results obtained from these interpolated data should prove more reliable than results obtained from a study of winds over a large area (such as in ref. 3). However, data below an altitude of 4,850 feet were not used since at lower altitudes data from other sources may be (and generally are) strongly influenced by local surface conditions.

Wind-speed profiles for Wallops Island obtained by the general bivariate normal (or binormal) distribution method are compared with corresponding profiles obtained by assuming the distributions to be circular and with profiles obtained by the empirical cumulative percentage frequency method. Wind-direction probability curves are compared for several altitudes. Component wind probability curves for a few selected directions are also presented.

SYMBOLS

A geometric description of the various symbols used herein is given in figure 1.

a	semimajor axis of ellipse (defined by eq. (A27))
A	constant in equation (A29)
b	semiminor axis of ellipse (defined by eq. (A28))
B	variable in equation (A29) (defined in eq. (A31))

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$c = \frac{\sigma_v}{\sigma_u}$

C	constant in equation (A25)
$f(\ )$	probability density function of designated variable or variables
G	constant defined by equation (A18)
K	constant in equation (A24)
N	total number of samples or wind observations in a distribution
P	probability
$P(\ )$	probability of designated variable or variables being within specified limits
R	radius of circle about origin of coordinate system or nondimensional velocity normalized by σ_u, V/σ_u
S	region in uv-plane or in $\xi\eta$-plane
u	wind-speed component along u (east-west) axis (zonal wind), positive when from east, fps
$\bar{u}$	mean east-west wind speed, fps
u'	wind-speed component along u'-axis, fps (eq. (A35))
$\bar{u}'$	mean wind component along u'-axis
v	wind-speed component along v (north-south) axis (meridional wind), positive when from north, fps
$\bar{v}$	mean north-south wind speed, fps
v'	wind-speed component along v'-axis, fps (eq. (A36))
$\bar{v}'$	mean wind component along v'-axis
V	magnitude of wind vector, fps
$\bar{V}$	magnitude of vector mean wind, $\sqrt{\bar{u}^2 + \bar{v}^2}$, fps

$\bar{W} = \frac{\bar{V}}{\sigma_u}$

x, y	arbitrary variables
$\sum x$	sum of all x from 1 to N, same units as quantity x
α	angle of rotation of u', v' axis system, positive counterclockwise from u (east-west) axis, deg
η	wind speed along semiminor axis of wind-distribution ellipse, fps
θ	direction from which wind blows, positive clockwise from north, deg
$\bar{\theta}$	direction from which vector mean wind blows, positive clockwise from north, $\tan^{-1} \frac{\bar{u}}{\bar{v}}$, deg
ξ	wind speed along semimajor axis of wind-distribution ellipse, fps
ρ	correlation coefficient of wind components
σ	standard deviation of wind components in a circular distribution, fps
σ_x	standard deviation of x, same units as quantity x
σ_{xy}	covariance of x and y, same units as product of x and y
ϕ	angular difference between mean vector wind and arbitrary wind, positive clockwise, $\theta - \bar{\theta}$, deg
ψ	angle of rotation of ξ, η axis from u, v axis, positive counterclockwise from u (east-west) axis, deg

Subscripts:

WAL	data for Wallops Island
ORF	data for Norfolk
DCA	data for Washington (observatory at Silver Hill, Md.)
i	arbitrary value, that is, 1, 2, . . .

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TREATMENT OF DATA

Assumptions

Use of the statistical analysis methods employed herein to predict probabilities involves certain major assumptions which are listed as follows along with the reasons for belief in their validity:

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(1) The population sample is sufficiently large to assure the validity of a large sample estimate of statistical parameters. Over 100 samples exist for all months and altitudes except at 86,150 feet. (See table I.) At this altitude the sample size approaches, in some instances, the limits of a statistically large sample.

(2) The orthogonal components of wind vectors are normally distributed variables. The basic data were not available in a form which permitted a rigorous evaluation of this assumption - that is, individual wind measurements were not available but only a statistical summary. However, the implications of this assumption are discussed in reference 3, and the assumption of normality is shown to be valid for most locations. The distribution generally is not normal at low altitudes in coastal regions (due to sea breezes) or in certain anomalous weather regions (hurricane regions, for example) as discussed in reference 3. It should be noted that the magnitudes of the wind vectors (that is, wind speeds) or the directions of the vectors generally are not normally distributed variables.

(3) Pressure altitudes are adequately defined by a standard-atmosphere geometric altitude. Since at a particular location a given pressure altitude varies over a small range of geometric altitude, the use of a standard atmosphere (ref. 5) should not introduce appreciable error in the final results.

(4) A straight-line fairing between points at the altitudes selected for obtaining wind distribution data gives a representative wind profile. Since the fine time varying structure of the atmosphere (gusts) is not within the scope of this investigation, the wind profiles obtained in this manner should describe adequately the macroscopic variations of the winds. Further details of typical wind-speed profiles are presented in references 6 and 7, and particular emphasis is given to the fine structure variation in reference 8.

(5) The statistical analysis of this report will be valid for future use. There is difficulty in making exact long-range predictions of weather phenomena from past observations. This is true of winds, particularly where extreme excursions of wind patterns have occurred, such as anomalous space and time variations of jet-stream behavior.

However, since the available data cover a reasonably long period of time and if the results are used to represent Wallops Island winds for periods no shorter than months or seasons (rather than what the wind will be at a specific time), the information reported herein should provide reliable estimates of the probabilities of occurrence of various wind parameters.

Data Source

Since sufficient wind sampling data were not available at Wallops L
Island, Va., measurements made at two adjacent geographical locations 2
(Weather Bureau observatory at Silver Hill, Maryland, and Norfolk, Va.) 0
were used as a basis for determining the Wallops Island wind distribu- 5
tions. The basic data for these two stations were processed and reduced 6
to statistical parameters by the National Weather Records Center, Asheville, North Carolina. These parameters were furnished as a monthly summary of statistical data for various pressure altitudes.

The basic data were obtained by use of ground-tracked balloon-borne radiosondes with either SCR-658 or AN/GMD-1 rawin systems. These systems are described in reference 9. The equipment used and the period of years covered in the statistical sampling are as follows:

Station	Period	Equipment
Norfolk, Va.	Dec. 1952 to June 1958	AN/GMD-1
Washington, D.C.	Nov. 1950 to Dec. 1952	SCR-658
Washington, D.C.	Jan. 1953 to Mar. 1958	AN/GMD-1

At both stations, the observations nominally were made four times daily at the following Greenwich mean times:

Prior to May 1957 . . . . 0300, 0900, 1500, and 2100
After May 1957 . . . . 0000, 0600, 1200, and 1800

The data were taken at the altitudes noted in table I.

Neither the original wind data nor the statistically processed data are available in published documents. However, the data can be obtained from the National Weather Records Center at Asheville, N.C.

Interpolation of Wind Data

In order to use the wind data from Washington and Norfolk in arriving at statistical wind distributions at Wallops Island, an interpolation scheme was required. Although it is common practice in meteorology to interpolate wind data between sampling stations to obtain data applicable at intermediate stations, such procedures require close examination.

A limited sampling of wind data was available for Wallops Island. These data were compared with data obtained at nearly the same time of day (early afternoon) at Washington and Norfolk. Approximately 100 comparisons were made with a randomly chosen set of samples covering a 1-year period. The wind vector at Wallops was found to lie about midway in value of magnitude and direction between the magnitude and direction of the winds at Norfolk and Washington. Geographical considerations and the results of reference 3 show that these results would be expected for these locations.

The basic parameters defining the statistical distribution for Wallops Island were determined by use of the following equations:

$$\sum u_{WAL} = \frac{\sum u_{DCA} + \sum u_{ORF}}{2} \tag{1}$$

$$\sum v_{WAL} = \frac{\sum v_{DCA} + \sum v_{ORF}}{2} \tag{2}$$

$$\sum (uv)_{WAL} = \frac{\sum (uv)_{DCA} + \sum (uv)_{ORF}}{2} \tag{3}$$

$$\sum (u^2)_{WAL} = \frac{\sum (u^2)_{DCA} + \sum (u^2)_{ORF}}{2} \tag{4}$$

$$\sum (v^2)_{WAL} = \frac{\sum (v^2)_{DCA} + \sum (v^2)_{ORF}}{2} \tag{5}$$

$$N_{WAL} = \frac{N_{DCA} + N_{ORF}}{2} \tag{6}$$

With the use of the basic parameters obtained from equations (1) to (6), the remaining quantities defining the wind distribution (such as mean wind speed, standard deviation, and correlation coefficient) were determined by the equations for these quantities given in the appendix. All the parameters determined for Wallops Island are given in table II.

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It should be noted that the basic parameters for Norfolk and Washington did not differ greatly; a typical set of values is shown in table III. Therefore, either the Norfolk or Washington data could have been applied directly to Wallops Island with little error; however, it is believed that the averaging process provided data which are more accurate. Also, the sample population is almost doubled by the procedure. The use of equations (1) to (6), of course, might appear to give more weight to the station having the larger sample of winds. However, more accurate calculations for typical cases showed that the difference was negligible because of the large sampling at both stations.

Data were available at two additional altitude levels at Washington, these altitudes being 34,055 and 38,735 feet. Since maximum wind speeds generally occur at altitudes in the vicinity of 30,000 to 40,000 feet at the latitude of Wallops Island, data from Washington were used directly to determine wind distributions at these altitudes for Wallops Island rather than (in the absence of data) attempting to fair an interpolated curve through this altitude interval.

The quantities referred to herein as parameters are, of course, the estimates of the parameters of the total infinite population, based on samples from the population, rather than the true population parameters.

Accuracy

Several potential sources of error exist in the determination of the wind vectors at altitude by use of rawinsonde systems. Unfortunately, there does not seem to be complete agreement among those who have studied the accuracy problem as to the magnitude of the possible system and data-reduction errors. It is known that under certain circumstances the magnitude of the wind speeds can have errors of 100 feet per second or more and that the directions can be 180° in error. Such occurrences are fairly rare. As noted in reference 10, the errors are a function of both the altitude of the balloon and the elevation angle from the ground receiving station to the radiosonde balloon with the major errors occurring at low elevation angles. The elevation angle of any individual measurement of the basic data used herein is not known. However, it is believed that for altitudes below 50,000 feet the data should have errors not exceeding the generally accepted values for rawinsonde data which are as follows:

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Wind speed (when speed <50 knots), knots	$\pm 2\frac{1}{2}$
Wind speed (when speed >50 knots), knots	± 5
Wind direction, deg	± 5

Above 50,000 feet, a generally accepted value of accuracy is ±10 percent for wind speed and $\pm 5^{\circ}$ for wind direction. At these high altitudes, errors are closely related to the bias problem, which is discussed subsequently; that is, the probability that high wind-speed data were not obtained at high altitude is more likely than the probability of occurrence of large measurement errors as such. It should be noted that, since the Washington and Norfolk data were obtained from independent observations, the close agreement of the various wind parameters at these two stations (typical values being shown in table III) tends to rule out the occurrence of any significant number of random errors in the basic data.

Bias

The rawinsonde balloon is tracked from a fixed ground station. Therefore, if the winds blow constantly at high speed from a fixed direction, as the balloon rises it may pass quickly below the horizon of the ground receiver station (or pass quickly beyond the lower elevation limits where ground clutter may interfere with the accurate reception of signals). Hence, high-speed winds at high altitudes may not be measured unless such winds occur in combination with low-speed winds at low altitude and/or appropriate direction changes.

Therefore, there can be a bias in the high-altitude wind data; this bias probably begins to have some effect on the data at altitudes as low as 25,000 to 30,000 feet. The effects of the bias are difficult to evaluate. It probably causes the high-altitude wind speeds to be underestimated by some unknown amount. It is known that in some wind samplings this bias has occurred.

The fact that the value of N for any month at extreme altitudes is generally lower in the winter months than in the summer months (for the Washington and Norfolk data) lends some confirmation to the probable existence of some bias. However, it should be recognized that the low values of N at high altitudes are partially due to various equipment failures such as balloon bursting at high altitudes after good low-altitude data have been obtained.

Computational Methods

The empirical cumulative percentage frequency wind-speed profiles (that is, considering wind speed as a function of altitude without regard to direction) presented herein were obtained by cumulating the winds for each month of the year at the various altitude levels (table I) for both Washington and Norfolk. These wind data were available only in 11 speed class intervals in knots, as follows: 1 to 9, 10 to 19, 20 to 29, 30 to 39, 40 to 49, 50 to 59, 60 to 74, 75 to 99, 100 to 149, 150 to 199, ≥ 200. The upper wind speed in each class was used for the class mark (that is, the value used in plotting wind speed as a function of frequency of occurrence). Empirical cumulative frequency of direction curves (that is, considering direction without regard to speed) were obtained in a similar fashion by cumulating the observations tabulated at each of the 16 principal compass directions; this amounts to using a class interval of $22\frac{1}{2}^{\circ}$ with the midpoint of the interval at the class mark.

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The wind-speed profiles and probability of direction curves for the elliptical binormal distribution were obtained by numerical integration of the elliptical distribution function on the IBM 7090 electronic data processing system by using Gauss's iteration formulas (ref. 11). The iteration was repeated until there was no change in the fourth decimal place of the probability values being computed. For wind-speed profiles, the integration was taken over the area of concentric circles centered at the origin of the coordinate system; for the probability of direction curves, the integration was taken over regions bounded by infinite radius vectors from the origin. A graphical illustration of the integration areas is presented in figure 1(b).

For the circular binormal distribution, general curves of wind-speed and wind-direction probabilities were generated by numerical integration of the circular distribution function on the IBM 7090 system and all data presented for this distribution were determined from plots of these general curves.

For the wind-speed profile envelopes along various component axes, the mean wind-speed component along these axes was first determined. To this mean value were added and subtracted the standard deviations of the distribution in directions parallel to these component axes. Both elliptical and circular distributions were considered.

Details of the statistical methods employed are presented in the appendix.

RESULTS AND DISCUSSION

Probability Wind-Speed Profiles

One of the parameters of interest in wind-distribution probability studies is the variation with altitude of wind speeds that will not be exceeded a given percentage of the time regardless of the direction from which the wind blows - that is, a probability wind-speed profile obtained by considering winds from all directions. Such profiles have been obtained for Wallops Island by using the three methods discussed in the section entitled "Computational Methods": the elliptical and circular binormal theoretical distribution methods and the empirical cumulative frequency method.

Comparisons of profiles obtained by these methods are shown in figures 2(a) to (c) for the months of January and July which are representative of the windiest and calmest months (or seasons) of the year at Wallops Island. The 50, 80, and 99 percent probability profiles determined by all three methods are in fair agreement for both January and July; in particular, the circular distribution profiles are in excellent agreement with those determined by considering the distribution to be elliptical. The most notable differences are in the 99 percent profiles determined by the empirical cumulative frequency method for January above 35,000 feet and for July below this altitude.

The 99.9 percent profiles determined by the elliptical and circular distribution methods are also in good agreement. There are, however, disagreements between the binormal profiles and the cumulative percentage frequency profiles for both months at several altitudes. The most significant differences occur in January above 40,000 feet; this is in an altitude region where bias is probably present in the data as noted in the section entitled "Bias."

Where class intervals are used to obtain wind-speed profiles, as is done in the empirical cumulative percentage frequency method, the error in determining probability increases with increasing class-interval size. In the probability regions which correspond to high wind speeds, the limited number of winds sampled can influence the empirical profiles considerably; whereas, theoretical profiles are much less affected by extreme values or by bias in the data. Therefore, it is believed that the wind profiles obtained by assuming the winds to follow an elliptical binormal distribution offer the best estimates of the true probabilities for Wallops Island, particularly for high probability profiles.

It should be noted, however, that, because of the accuracy limitations of the basic data and because of the unknown amount of bias in the data, caution should be observed when considering the use of wind-speed profiles above the 99 percent probability level. This limitation would be expected to apply generally to wind data obtained from the type of rawinsonde equipment used to gather the data considered herein.

Since extensive computing is required to generate wind-speed profiles for the elliptical distribution profiles, Wallops Island profiles determined by this method are presented for the months of January, April, July,
and October only in figures 3(a) to (d) for 50, 80, 90, 95, 99, and L
99.9 percent profiles. It has been shown (fig. 2) that the assumption 2
of a circular binormal distribution gives a reasonable representation 0
of wind-speed profiles for Wallops Island. As shown in the appendix, 5
relatively few parameters are required to define a generalized set of 6
curves for wind-speed probability profiles when circularity is assumed. Such a set of curves is presented in figure 4.

If values of $\bar{V}$ and σ are known, figure 4 can be used to determine either the probability of occurrence of a wind of magnitude V or the magnitude of wind V which will correspond to a particular probability.

Table II gives values of σ and $\bar{V}$ for Wallops Island which can be used with figure 4 to obtain probability profiles for any month of the year at any desired probability level. Calculated profiles (not presented herein) based on this procedure have shown that for Wallops Island all the wind speeds for the months of December, March, and February are about the same level as the January wind speeds (the windiest month). The November wind speeds are somewhat lower than those of this group and September wind speeds are still lower. The lowest wind speeds occur in July, with June and August wind speeds being generally of the same level; April wind speeds are higher, as shown in figure 3(b), and May wind speeds are slightly higher than April wind speeds. The October wind speeds present an interesting anomaly as they are generally lower than those of April as is shown in figure 3(d).

The curves of figure 4 can also be used to determine wind-speed probability profiles at any location where $\bar{V}$ and σ have been determined and where the assumption of a circular distribution is deemed valid.

Wind-Direction Probabilities

In a manner analogous to that used to obtain the wind-speed probabilities, wind-direction probability curves can be obtained; these curves include all wind speeds. Presented in figures 5(a) to (c) is a limited set of comparison curves for Wallops Island wind-direction probabilities as determined by the elliptical and circular binormal distribution methods and the empirical cumulative frequency methods.

On these curves $P(\theta_i) = P(0 \leqq \theta \leqq \theta_i)$. If $P(\theta_1 \rightarrow \theta \rightarrow \theta_2)$ is defined as the probability that θ lies between θ_1 and θ_2 as θ increases clockwise from θ_1 to θ_2, the probability over any region can be found as follows:

$$P(\theta_1 \rightarrow \theta \rightarrow \theta_2) = P(\theta_2) - P(\theta_1) \qquad (\theta_1 < \theta_2) \qquad (7)$$

$$P(\theta_1 \rightarrow \theta \rightarrow \theta_2) = 1 + \left[P(\theta_2) - P(\theta_1)\right] \qquad (\theta_1 > \theta_2) \qquad (8)$$

Since an extensive set of curves would be required for a complete description of direction probabilities for Wallops Island, only a limited, but typical, sampling is presented. However, the main conclusion to be drawn from these curves is not the probability values but the fact that the three methods of determining probability of direction are in reasonable agreement. In particular, the circular distribution results are in excellent agreement with the elliptical distribution results. This agreement also held for several other altitudes and months for which data have not been shown. Therefore, it seems reasonable to use circular distribution methods for determining direction probabilities for Wallops Island winds.

A set of generalized circular-wind-distribution probability of direction curves is presented in figure 6. These curves can be used with the data of table II to determine data for Wallops Island; the curves also are applicable for any location where the winds are distributed circularly. The method of using these curves is similar to that previously discussed for figures 4 and 5.

These curves show that, when the mean vector wind speed is low, winds from any direction are likely to occur. If the mean vector wind is reasonably high, there is increasing probability that the wind will blow from the direction of the mean vector wind $\bar{\theta}$.

Component Wind Probability

The results presented in the previous subsections are wind-speed profiles considering all directions and wind directions considering all speeds. Although this type of probability information is often considered of prime interest, the problem of what wind speeds are within given probability limits in a given direction is also of considerable importance. This probability distribution is often referred to as the component wind probability envelope and is defined by the statistical marginal distribution along various sets of orthogonal axes - that is, the distribution includes all wind-speed components along these axes.

Presented in figure 7 is a comparison of component wind envelopes for Wallops Island for the month of January in the east-west and northeast-southwest directions; "three sigma" profiles are compared. The agreement between the circular distribution envelopes and the elliptical distribution envelopes is good in both the east-west and the northeast-southwest directions. Therefore, it is believed that the assumption of circularity would be valid when determining Wallops Island component winds. However, the problem of determining component winds is relatively simple even though the elliptical distribution method is used; in fact, it is about as simple as using the circular distribution method. The procedures to be used are outlined in the appendix.

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Typical elliptical component wind profiles for one, two, and three sigma probabilities for Wallops Island are presented in figure 8. Data are shown for January in the east-west, north-south, northeast-southwest, and northwest-southeast directions and for July in the east-west and north-south directions. These data were determined by using the data of table II and the equations given in the appendix. By use of these tabulated data, component winds can be determined readily for any direction or month of interest. The method used, of course, applies to any location for which the appropriate statistical parameters have been determined.

Note that, in general, one cannot obtain wind-speed probability profiles, such as those of figure 3, directly from component wind-speed envelopes, such as those of figure 8.

CONCLUDING REMARKS

Statistical parameters which define wind probabilities at the NASA Wallops Station in an altitude range from 4,850 to 86,150 feet have been determined. The data were determined from measurements made with rawinsonde systems at Washington, D.C., and Norfolk, Va.

Wind-speed probabilities, obtained by considering winds from all directions, and wind-direction probabilities, obtained by considering all wind speeds, were determined for the NASA Wallops Station by three methods: the empirical cumulative frequency method and integration of the elliptical and circular binormal distribution functions over appropriate regions. It was concluded that the elliptical distribution method gave the best results for determining wind-speed and wind-direction probabilities; however, it was shown that the circular distribution method gave an adequate representation for these probabilities. General charts are presented which can be used with tabulated values of statistical parameters for Wallops Island to determine wind-speed and wind-direction probabilities by the circular distribution method; these charts can also be used at any location for which the appropriate statistical parameters are available and for which the assumption of circularity is valid.

Component wind-speed probabilities determined by the elliptical and circular distribution methods were compared. It was shown that the assumption of circularity was also valid for component winds. A method is discussed for obtaining component winds simply for an elliptical distribution.

The general mathematical statistics methods used are summarized in an appendix.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 23, 1962.

APPENDIX

MATHEMATICAL STATISTICS PROCEDURES

General

The salient points of the statistical methods used in this report are outlined here for reference. The discussion is slanted toward wind data, but the basic methods are applicable to studies of other variables. A more detailed discussion of some of these statistical methods can be found in several texts on statistics, references 12, 13, and 14 being typical examples.

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Wind Vectors

It is convenient to consider the east-west and north-south components of a group of winds observed at some particular altitude as statistical samples of winds. The following parameters can then be readily determined:

Mean east-west wind:

$$\bar{u} = \frac{\sum u}{N} = \frac{\sum |V| \sin \theta}{N} \tag{A1}$$

Mean north-south wind:

$$\bar{v} = \frac{\sum v}{N} = \frac{\sum |V| \cos \theta}{N} \tag{A2}$$

Standard deviation of east-west winds:

$$\sigma_u = \sqrt{\frac{1}{N} \sum (u - \bar{u})^2} = \sqrt{\frac{\sum (u)^2}{N} - \left(\frac{\sum u}{N}\right)^2} \tag{A3}$$

Standard deviation of north-south winds:

$$\sigma_v = \sqrt{\frac{1}{N} \sum (v - \bar{v})^2} = \sqrt{\frac{\sum (v)^2}{N} - \left(\frac{\sum v}{N}\right)^2} \tag{A4}$$

These parameters have a valid meaning regardless of the sample distribution but, since wind is a vector quantity, it is useful to define the additional statistical quantities associated with a distribution of winds:

Magnitude of mean vector wind:

$$\bar{V} = \sqrt{\bar{u}^2 + \bar{v}^2} \tag{A5}$$

Direction of mean vector wind:

$$\bar{\theta} = \tan^{-1} \frac{\bar{u}}{\bar{v}} \tag{A6}$$

Standard vector deviation of winds:

$$\sigma_V = \sqrt{\sigma_u^2 + \sigma_v^2} = \sqrt{\frac{\sum (V)^2}{N} - \bar{V}^2} \tag{A7}$$

If a distribution of the wind vectors is assumed, the foregoing parameters can be used in vector statistics methods to determine probability estimates of wind speeds and direction and various other information useful in rocket vehicle design as shown in the following sections.

Probability Density Functions

The probability density function (or frequency function) $f(x)$ associated with a random variable x defines the density or concentration of that variable throughout the region of the distribution of the variable. Then,

$$dP(x) = f(x)dx \tag{A8}$$

gives the probability that x will be found in an infinitesimal region of the distribution at x. The probability that x will be found in an interval between x_1 and x_2 is given by the following equation:

$$P(x) = P\left(x_1 \leqq x \leqq x_2\right) = \int_{x_1}^{x_2} f(x)dx \tag{A9}$$

The function $f(x)$ must be such that

$$0 \leqq \int_{x_1}^{x_2} f(x)dx \leqq 1 \tag{A10}$$

$$\int_{-\infty}^{\infty} f(x)dx \equiv 1 \tag{A11}$$

Note that equation (A9) is identically zero if $x_1 = x_2$; that is, this equation will not predict the probability of the occurrence of a specific value of x but only the probability that x will be found in a specified interval.

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If $f(x,y)$ is the joint probability density function of the independent random variables x and y, the probability that x and y will be found in a region $\left(x_1 \leqq x \leqq x_2;\ y_1 \leqq y \leqq y_2\right)$ is given by

$$P(x,y) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x,y)dx\ dy \tag{A12}$$

Wind Distribution Functions

If the vector wind components u and v are independent and are normally distributed, then the east-west and north-south wind components are each distributed according to the univariate Gaussian distribution as follows:

$$f(u) = \frac{1}{\sqrt{2\pi}\sigma_u} e^{-\frac{1}{2}\left(\frac{u-\bar{u}}{\sigma_u}\right)^2} \tag{A13}$$

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{1}{2}\left(\frac{v-\bar{v}}{\sigma_v}\right)^2} \tag{A14}$$

Then, under these conditions, the wind components will have the following joint probability density function which is the bivariate normal density distribution:

$$f(u,v) = f(u)f(v) = \frac{1}{2\pi\sigma_u\sigma_v} e^{-\frac{1}{2}\left[\left(\frac{u-\bar{u}}{\sigma_u}\right)^2 + \left(\frac{v-\bar{v}}{\sigma_v}\right)^2\right]} \tag{A15}$$

The probability that a wind vector, terminating at the origin of the coordinate axes, will originate in a region S of the uv-plane is given by

$$P(u,v) = \frac{1}{2\pi\sigma_u\sigma_v} \iint_S e^{-\frac{1}{2}\left[\left(\frac{u-\bar{u}}{\sigma_u}\right)^2 + \left(\frac{v-\bar{v}}{\sigma_v}\right)^2\right]} du\ dv \tag{A16}$$

If the component winds are not distributed independently, then equation (A16) must be modified to describe the probability and becomes

$$P(u,v) = \frac{1}{2\pi\sigma_u\sigma_v\sqrt{1-\rho^2}} \iint_S e^{-\frac{G}{2}} du\ dv \tag{A17}$$

where

$$G = \frac{1}{1-\rho^2}\left[\left(\frac{u-\bar{u}}{\sigma_u}\right)^2 - \frac{2\rho(u-\bar{u})(v-\bar{v})}{\sigma_u\sigma_v} + \left(\frac{v-\bar{v}}{\sigma_v}\right)^2\right] \tag{A18}$$

The quantity ρ is the correlation coefficient and describes the degree of interdependence between u and v as follows:

$$\rho = \frac{\sigma_{uv}}{\sigma_u\sigma_v} = \frac{\sum (uv) - \frac{\sum u \sum v}{N}}{N\sigma_u\sigma_v} \tag{A19}$$

The correlation coefficient can assume values of -1 through 1. A value of 1 indicates that with an increase in wind speed along one axis there will be an increase in speed along the other axis. If ρ is -1, the wind will decrease in speed along one axis as it increases along the other axis. If $\rho = 0$, there is no correlation; a fractional value of ρ indicates partial correlation.

For the Wallops Island winds, ρ in general was small but not zero. There are various standard tests to determine whether a fractional value of ρ is statistically significant; these tests indicated statistical significance for the values of ρ for Wallops Island winds. (The total sample N was used in making these tests.) However, it was shown in the section "Results and Discussion" of the text that the effect of correlation was negligible since the assumption of circularity was valid and circularity requires that $\rho = 0$.

Probability Ellipses

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For constant values of G, equation (A18) defines a family of homothetic ellipses (common origin, common axes, and constant eccentricity) in the uv-plane. If the origin of the u,v axis system is translated to a point $\bar{u},\bar{v}$ and then rotated through the angle

$$\psi = \frac{1}{2}\tan^{-1}\left(\frac{2\rho\sigma_u\sigma_v}{\sigma_u^2 - \sigma_v^2}\right) \tag{A20}$$

it can be shown that in this new coordinate system ξ,η (fig. 1) the probability can be written as follows:

$$P(u,v) = P(\xi,\eta) = \frac{1}{2\pi\sigma_\xi\sigma_\eta}\iint_S e^{-\frac{1}{2}\left[\left(\frac{\xi-\bar{\xi}}{\sigma_\xi}\right)^2+\left(\frac{\eta-\bar{\eta}}{\sigma_\eta}\right)^2\right]}d\xi\,d\eta \tag{A21}$$

This translation and rotation defines a new set of axes for which $\rho = 0$.

Since equations (A17) and (A21) are equal over the same region of integration, it can be shown that

$$\sigma_\xi\sigma_\eta = \sigma_u\sigma_v\sqrt{1 - \rho^2} \tag{A22}$$

and

$$\sigma_\xi^2 + \sigma_\eta^2 = \sigma_u^2 + \sigma_v^2 \tag{A23}$$

The standard deviations σ_ξ and σ_η can be found in terms of σ_u and σ_v by solving the following determinant for the absolute values of K:

$$\begin{vmatrix} \sigma_u{}^2 - K^2 & \sigma_u \sigma_v \rho \\ \sigma_u \sigma_v \rho & \sigma_v{}^2 - K^2 \end{vmatrix} = 0 \tag{A24}$$

where

σ_ξ largest value of K

σ_η smallest value of K

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If $G = C^2 =$ Constant, equation (A18) which defines a family of homothetic ellipses can be written

$$\left(\frac{\xi}{\sigma_\xi}\right)^2 + \left(\frac{\eta}{\sigma_\eta}\right)^2 = G = C^2 \tag{A25}$$

and the semiaxes of any one of these ellipses become $C\sigma_\xi$ and $C\sigma_\eta$.

The differential area of an ellipse given by equation (A25) is $2\pi C\sigma_\xi\sigma_\eta \, dC$ so that equation (A21) can be written

$$P(\xi,\eta) = \int_0^C e^{-\frac{C^2}{2}} C \, dC = 1 - e^{-\frac{C^2}{2}} \tag{A26}$$

Equation (A26) gives the probability that a wind vector will originate inside an ellipse given by equation (A25) and will terminate at the origin $u = v = 0$. Thus, for any probability P the semiaxes of the probability ellipse are given by the following equations:

$$a = \left[2 \log_e\left(\frac{1}{1 - P}\right)\right]^{1/2} \sigma_\xi \tag{A27}$$

$$b = \left[2 \log_e\left(\frac{1}{1 - P}\right)\right]^{1/2} \sigma_\eta \tag{A28}$$

Hence, if a normally distributed set of wind vectors is observed, it will be found that the vectors originating from a point on or within an ellipse centered at $\bar{u},\bar{v}$ with semimajor and semiminor axes given by $C\sigma_\xi$ and $C\sigma_\eta$ will have a probability P of occurrence (where C

and P are related as shown in eq. (A26)). The major axis of this ellipse will be rotated at an angle ψ with respect to the u (east-west) axis. Obviously, if $\sigma_u = \sigma_v = \sigma$ and if $\rho = 0$, then the ellipse becomes a circle. Therefore, the probability distributions defined by equations (A16) and (A17) are referred to as circular and elliptical binormal distributions, respectively.

Wind-Speed and Wind-Direction Probability Profiles

Wind-speed and wind-direction probabilities are found by integration of equation (A17) over appropriate regions. The integration is simpler if the following substitutions are made: L 2 0 5 6

$$u = V \sin\theta \qquad \bar{u} = \bar{V} \sin\bar{\theta}$$

$$v = V \cos\theta \qquad \bar{v} = \bar{V} \cos\bar{\theta}$$

$$R = \frac{V}{\sigma_u} \qquad \bar{W} = \frac{\bar{V}}{\sigma_u}$$

$$c = \frac{\sigma_v}{\sigma_u}$$

Then equation (A17) becomes

$$P\left(R_1 \leqq R \leqq R_2,\ \theta_1 \leqq \theta \leqq \theta_2\right) = A \int_{\theta_1}^{\theta_2} \int_{R_1}^{R_2} e^{-\frac{B}{2\left(1-\rho^2\right)}} R\, dR\, d\theta \tag{A29}$$

where

$$A = \frac{1}{2\pi c} \frac{1}{\sqrt{1 - \rho^2}} \tag{A30}$$

and

$$B = R^2\left(\sin^2\theta + \frac{\cos^2\theta}{c^2}\right) + \bar{W}^2\left(\sin^2\bar{\theta} + \frac{\cos^2\bar{\theta}}{c^2}\right) - 2R\bar{W}\left(\sin\theta \sin\bar{\theta} + \frac{\cos\theta \cos\bar{\theta}}{c^2}\right) - \frac{2\rho}{c}\left[R^2 \sin\theta \cos\theta + \bar{W}^2 \sin\bar{\theta} \cos\bar{\theta} - R\bar{W} \sin\left(\theta + \bar{\theta}\right)\right] \tag{A31}$$

For the special case of the circular distribution, equation (A29) reduces to

$$P\left(R_1 \leqq R \leqq R_2,\ \phi_1 \leqq \phi \leqq \phi_2\right) = \frac{1}{2\pi} \int_{\phi_1}^{\phi_2} \int_{R_1}^{R_2} e^{-\frac{1}{2}\left(R^2+\overline{W}^2-2R\overline{W}\cos\ \phi\right)} R\ dR\ d\phi \tag{A32}$$

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where $\sigma_u = \sigma_v = \sigma$ and $\phi = \theta - \bar{\theta}$.

If equation (A29) or equation (A32) is integrated over a region described by a circle around the origin of the u,v coordinate axes (with the origin of the distribution at $\bar{u},\bar{v}$), then the resulting probability will be that of a wind vector originating on or within this circle and terminating at $u = v = 0$ (including winds from all directions). The integration limits of equation (A29) are then $\theta_1 = 0$ and $\theta_2 = 2\pi$ and $R_1 = 0$ and $R_2 = R_2$. Since the probability is a function of the variable R_2, this integral is solved by substituting the desired value of probability into the left-hand side and a value of R_2 which satisfies equation (A29) is found by iteration. For equation (A32) a similar procedure is followed with $\phi_1 = 0$ and $\phi_2 = 2\pi$ and $R_1 = 0$ and $R_2 = R_2$. For the elliptical distribution (eq. (A29)), the probability is a function of $\left(V/\sigma_u,\ \overline{V}/\sigma_u,\ \bar{\theta},\ c,\ \rho\right)$ and, therefore, it is not practical to generate a table or set of curves giving general solutions. However, for the circular distribution (eq. (A32)), a simple table can be constructed for P as a function of $\overline{V}/\sigma$ and R/σ; a plot of such a table is presented in figure 4.

In order to find the probability that a wind (including all wind speeds) will blow from a region bounded by $\theta = 0$ and $\theta = \theta_2$ (or $\phi = 0$ and $\phi = \phi_2$), the limits of integration of equation (A29) are changed to $\theta_1 = 0$ and $\theta_2 = \theta_2$ and $R_1 = 0$ and $R_2 \rightarrow \infty$ and of equation (A32) to $\phi_1 = 0$ and $\phi_2 = \phi_2$ and $R_1 = 0$ and $R_2 \rightarrow \infty$. In performing the numerical integration, the upper limit of R was, of course, limited to a finite but high value. The elliptical binormal distribution again requires an extensive general table, and a particular solution for a given set of variables is all that is generally practical. The circular distribution once more reduces to a sufficiently small set of variables with P depending on ϕ and $\overline{V}/\sigma$, and a general solution can be given as was presented in figure 6. The probability of a wind blowing from a region bounded by radius vectors having angles of θ_1 and θ_2 (or ϕ_1 and ϕ_2) is

$$P(\theta_2) - P(\theta_1) \qquad (\theta_2 > \theta_1)$$

and

$$1 + \left[P(\theta_2) - P(\theta_1)\right] \qquad (\theta_1 > \theta_2)$$

Obviously, the probability of a wind of a given magnitude coming from a specified range of direction can also be found by using equations (A29) and (A32). However, this probability value is generally not as useful as the probabilities associated with component winds as discussed in the following section.

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Component Wind Probability Envelopes

The projection of the wind distribution function on an axis gives the marginal distribution of the wind vectors along that axis; for example, the marginal distribution of u defines the probability of occurrence of the east-west component of wind u having the value u_i regardless of the value of v. In practice, one is usually interested in the marginal distributions of a set of orthogonal components, such as down-range and cross-range components. The marginal density or frequency functions for u and v are

$$f(u) = \frac{1}{\sqrt{2\pi}\sigma_u} e^{-\frac{1}{2}\left(\frac{u-\bar{u}}{\sigma_u}\right)^2} \tag{A33}$$

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-\frac{1}{2}\left(\frac{v-\bar{v}}{\sigma_v}\right)^2} \tag{A34}$$

Equations (A33) and (A34) are identical with equations (A13) and (A14); that is, the marginal distributions are merely the univariate normal distributions along the axes of interest. The integral of this function is given in any standard mathematical table. Therefore, probability envelopes expressed in standard deviations from the wind speed can be determined very simply; for example, 99.73 percent of all east-west winds will be within a value of plus or minus three standard deviations of the mean east-west wind $\bar{u}$.

For a set of axes u' and v' rotated through an angle α from the east-west axis, the wind component probability envelopes are defined by addition of multiples of the standard deviations along these (u',v') axes to the value of the mean wind along these axes. Rotating the axes gives

$$u' = u \cos \alpha + v \sin \alpha \tag{A35}$$

$$v' = v \cos \alpha - u \sin \alpha \tag{A36}$$

and

$$\bar{u}' = \bar{u} \cos \alpha + \bar{v} \sin \alpha \tag{A37}$$

$$\bar{v}' = \bar{v} \cos \alpha - \bar{u} \sin \alpha \tag{A38}$$

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Since

$$\sigma_u = \sqrt{\frac{1}{N} \sum (u - \bar{u})^2} \tag{A39}$$

then,

$$\sigma_{u'} = \sqrt{\frac{1}{N} \sum \left[(u - \bar{u})\cos \alpha + (v - \bar{v})\sin \alpha\right]^2}$$

which reduces to

$$\sigma_{u'} = \sqrt{\sigma_u^2 \cos^2\alpha + \sigma_v^2 \sin^2\alpha + 2\rho\sigma_u\sigma_v \sin \alpha \cos \alpha} \tag{A40}$$

Similarly,

$$\sigma_{v'} = \sqrt{\sigma_v^2 \cos^2\alpha + \sigma_u^2 \sin^2\alpha - 2\rho\sigma_u\sigma_v \sin \alpha \cos \alpha} \tag{A41}$$

For the circular distribution, the mean values are the same as those given by equations (A37) and (A38) and the standard deviation is simply σ. If a distribution has small ellipticity, a circular distribution standard deviation may be approximated by

$$\sigma = \sqrt{\frac{\sigma_u^2 + \sigma_v^2}{2}} \tag{A42}$$

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TABLE I.- NUMBER OF WIND OBSERVATIONS AT NORFOLK, VA., AND AT WASHINGTON, D.C.[1]

Altitude, ft	Washington, D.C.		Norfolk, Va.	
	Max. N	Min. N	Max. N	Min. N
4,850	779	698	730	566
9,850	758	687	739	577
19,000	716	601	725	572
31,450	673	463	684	532
34,055	670	421	---	---
38,735	660	386	---	---
46,350	649	331	580	430
52,800	635	303	472	295
66,670	549	221	275	183
79,650	433	168	237	100
86,150	239	72	82	20

[1]Data shown are a summary of the total number of observations for 1 month for the entire data period for the months having the maximum and the minimum number of observations.

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TABLE II.- STATISTICAL PARAMETERS FOR WIND DISTRIBUTIONS

AT THE NASA WALLOPS STATION

Altitude, ft	$\bar{u}$, fps	$\bar{v}$, fps	σ_u, fps	σ_v, fps	σ_ξ, fps	σ_η, fps	ρ	ψ, deg	$\bar{V}$, fps	$\bar{\theta}$, deg	σ_V, fps	σ, fps	$\frac{\bar{V}}{\sigma}$
January													
4,850	-28.3	7.2	23.7	29.4	30.9	21.7	0.27	64.2	29	284	37.8	26.7	1.09
9,850	-48.4	4.3	27.7	32.1	33.6	25.9	.21	62.6	49	275	42.4	30.0	1.63
19,000	-82.3	4.3	44.6	47.5	48.6	43.5	.09	62.6	82	273	65.2	46.1	1.78
31,450	-124.0	5.3	65.7	66.4	66.7	65.3	-.02	119.5	124	272	93.4	66.1	1.88
34,055	-123.0	5.9	67.4	67.6	70.6	64.2	-.09	134.0	123	273	95.5	67.5	1.82
38,735	-118.5	3.0	57.2	55.0	57.3	55.0	-.01	170.0	119	271	79.4	56.1	2.12
43,650	-116.2	1.5	50.3	41.5	50.3	41.5	-.02	176.9	116	271	65.2	46.1	2.52
52,800	-80.3	-.6	36.2	27.3	36.2	27.2	-.04	3.8	80	270	45.3	32.1	2.50
66,670	-34.9	1.8	29.9	18.6	30.0	18.5	.04	2.4	35	273	35.2	24.9	1.41
79,650	-34.6	1.8	31.5	20.8	32.2	19.8	.24	14.5	35	273	37.7	26.7	1.31
86,150	-47.3	1.2	36.6	25.4	36.8	25.2	.09	7.1	47	271	44.6	31.5	1.49
February													
4,850	-30.6	0.9	22.6	29.2	29.5	22.3	0.11	78.9	31	272	36.9	26.1	1.19
9,850	-50.5	.4	24.4	30.1	30.2	24.3	.06	82.6	50	270	38.8	27.4	1.82
19,000	-83.4	-1.3	39.0	42.6	42.7	39.0	.01	85.8	83	269	57.8	40.8	2.03
31,450	-119.3	-2.1	52.6	61.1	61.3	52.4	.04	82.7	119	269	80.6	57.0	2.09
34,055	-122.1	.7	57.1	55.1	55.1	52.5	-.02	100.0	123	270	87.1	61.6	1.98
38,735	-122.7	2.6	52.5	59.1	68.3	56.2	-.16	152.0	132	271	76.1	53.8	2.29
43,650	-119.3	1.2	42.8	39.6	42.8	39.5	-.02	174.1	119	271	58.3	41.2	2.89
52,800	-87.9	-.8	34.4	27.5	34.6	27.3	-.07	171.1	88	270	44.0	31.1	2.83
66,670	-35.7	.1	29.9	19.2	29.9	19.1	.04	2.7	36	270	35.5	25.1	1.43
79,650	-17.0	-3.1	32.6	18.8	32.6	18.6	.10	5.0	17	260	37.6	26.6	.64
86,150	-6.3	.6	20.1	8.3	20.1	8.3	.10	2.7	6	275	21.7	15.3	.39
March													
4,850	-28.4	2.9	23.2	27.4	27.4	23.2	0.01	88.9	29	276	35.9	25.4	1.14
9,850	-47.5	3.4	27.0	29.4	29.8	26.7	.07	71.1	48	274	39.9	28.2	1.70
19,000	-80.2	3.5	40.7	39.8	41.3	39.1	.05	33.3	80	272	56.9	40.3	1.99
31,450	-124.4	3.2	57.6	57.9	58.8	56.7	-.04	131.0	124	271	81.7	57.8	2.15
34,055	-131.5	4.3	65.9	59.1	68.3	56.2	-.16	152.0	132	272	88.5	62.6	2.11
38,735	-128.2	6.0	60.3	53.1	61.5	51.7	-.12	159.0	128	273	80.3	56.8	2.25
43,650	-116.7	4.3	44.4	43.2	47.8	39.4	-.19	139.2	117	272	61.9	43.8	2.67
52,800	-81.3	1.8	35.6	28.0	36.6	26.7	-.20	160.3	81	271	45.3	32.0	2.53
66,670	-28.9	.1	31.1	15.4	31.1	15.3	-.08	176.9	29	270	34.7	24.5	1.18
79,650	-17.3	2.1	31.3	14.7	31.4	14.5	.14	4.7	17	277	34.6	24.5	.70
86,150	-19.2	3.1	30.6	17.0	30.6	17.0	.04	1.7	19	279	35.0	24.8	.77

TABLE II.- STATISTICAL PARAMETERS FOR WIND DISTRIBUTIONS

AT THE NASA WALLOPS STATION - Continued

Altitude, ft	$\bar{u}$, fps	$\bar{v}$, fps	σ_u, fps	σ_v, fps	σ_ξ, fps	σ_η, fps	ρ	ψ, deg	$\bar{V}$, fps	$\bar{\theta}$, deg	σ_V, fps	σ, fps	$\frac{\bar{V}}{\sigma}$
April													
4,850	-25.0	0.2	22.9	15.5	27.3	21.5	0.19	62.3	25	270	27.7	19.6	1.28
9,850	-37.3	2.6	26.6	16.7	29.7	24.9	.16	54.6	37	274	31.4	22.2	1.67
19,000	-57.8	4.5	37.6	20.7	38.3	34.2	.09	24.9	58	274	42.9	30.4	1.91
31,450	-84.1	9.2	54.1	30.3	54.3	51.0	.03	167.5	85	276	62.0	43.9	1.94
34,055	-87.9	11.5	57.2	53.6	57.7	53.1	-.06	159.0	89	277	78.4	55.4	1.61
38,735	-89.7	13.4	52.8	52.5	48.5	52.8	-.05	166.0	91	278	74.5	52.7	1.73
43,650	-82.9	12.4	44.2	20.7	44.2	34.9	.02	2.3	84	278	53.2	37.6	2.23
52,800	-54.9	8.2	29.7	13.4	29.7	22.6	.03	3.2	56	278	32.6	23.0	2.43
66,670	-12.8	4.0	22.3	8.0	22.3	13.5	.01	.7	13	287	23.7	16.8	.78
79,650	-7.6	1.5	20.5	6.9	20.5	11.6	.01	.6	8	281	21.6	15.3	.52
86,150	-16.3	.9	24.1	7.4	24.3	12.2	.02	8.1	16	273	25.2	17.8	.90
May													
4,850	-19.3	2.9	19.5	19.0	20.6	17.8	0.15	40.1	20	278	27.2	19.3	1.04
9,850	-30.1	5.7	22.3	23.1	24.1	21.3	.12	52.4	31	281	24.2	17.1	1.82
19,000	-47.2	6.1	30.7	32.2	33.1	29.7	.10	57.0	48	277	44.5	31.5	1.53
31,450	-68.7	4.1	45.2	48.1	51.4	41.5	.20	53.6	69	273	66.0	46.7	1.48
34,055	-74.9	7.7	49.6	54.2	57.1	46.3	.19	58.0	75	276	73.5	52.0	1.44
38,735	-76.1	9.8	50.2	53.4	56.2	47.0	.17	56.0	77	277	73.2	51.8	1.49
43,650	-70.6	8.0	40.2	39.4	43.1	36.2	.17	42.0	71	276	56.3	39.8	1.78
52,800	-39.7	6.1	25.6	23.8	27.1	22.1	.18	34.2	40	279	35.0	24.7	1.62
66,670	-30.1	2.7	15.4	10.7	15.6	10.4	.15	10.9	4	311	18.8	13.3	.30
79,650	2.8	.7	16.8	9.3	16.8	9.2	.07	3.3	3	76	19.2	13.6	.22
86,150	-3.2	-.4	22.8	10.6	22.8	10.6	.05	1.8	3	262	25.1	17.8	.17
June													
4,850	-12.5	3.1	17.4	16.8	18.4	15.7	0.15	37.5	13	283	24.2	17.1	0.76
9,850	-18.8	5.9	21.2	20.7	22.2	19.6	.12	39.0	20	287	29.6	21.0	.95
19,000	-29.4	9.3	29.1	29.3	31.6	26.5	.17	46.2	31	287	41.3	29.2	1.06
31,450	-43.3	12.1	43.3	48.2	51.2	39.8	.22	58.0	45	286	64.8	45.8	.98
34,055	-44.1	17.0	43.4	53.3	54.8	41.4	.19	69.0	47	291	68.7	48.6	.97
38,735	-46.9	20.3	44.1	51.8	53.8	41.6	.20	65.0	51	293	68.0	48.1	1.06
43,650	-44.5	16.5	44.3	41.1	47.7	37.0	.24	36.3	47	290	60.4	42.7	1.10
52,800	-19.1	11.7	26.0	21.5	26.9	20.4	.20	22.9	22	301	33.7	23.9	.92
66,670	14.2	2.4	13.3	9.1	13.3	9.1	-.01	177.2	14	80	16.1	11.4	1.23
79,650	23.2	.2	12.8	9.7	12.8	9.7	.00	179.6	23	89	16.1	11.4	2.03
86,150	8.2	.2	14.7	5.8	14.8	5.8	.06	1.6	8	89	15.8	11.2	7.16

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TABLE II.- STATISTICAL PARAMETERS FOR WIND DISTRIBUTIONS

AT THE NASA WALLOPS STATION - Continued

Altitude, ft	$\bar{u}$, fps	$\bar{v}$, fps	σ_u, fps	σ_v, fps	σ_ξ, fps	σ_η, fps	ρ	ψ, deg	$\bar{V}$, fps	$\bar{\theta}$, deg	σ_V, fps	σ, fps	$\frac{\bar{V}}{\sigma}$
							July						
4,850	-11.8	3.5	16.4	13.8	17.1	12.9	0.22	25.5	12	287	21.4	15.2	0.79
9,850	-18.3	6.2	18.9	14.6	19.1	14.4	.12	12.2	19	289	23.9	16.9	1.12
19,000	-28.2	8.8	25.6	20.5	25.9	20.0	.13	14.6	30	287	36.5	25.8	1.16
31,450	-40.3	12.1	39.6	37.6	42.9	33.7	.23	38.6	42	287	54.6	38.6	1.09
34,055	-48.0	15.4	45.2	44.7	50.5	38.6	.26	44.0	50	288	63.6	45.0	1.11
38,735	-49.1	17.9	47.8	46.9	52.6	41.4	.23	43.0	52	290	67.0	47.4	1.10
43,650	-38.2	18.0	40.7	36.7	43.2	33.7	.22	32.5	42	295	54.8	38.8	1.08
52,800	-11.1	11.1	24.2	18.3	24.4	18.0	.12	11.4	16	315	30.3	21.5	.75
66,670	22.4	1.6	11.5	8.3	11.6	8.2	.12	10.0	22	86	14.2	10.0	2.19
79,650	36.6	-.8	10.8	9.3	10.8	9.3	-.05	171.4	37	91	14.3	10.1	3.67
86,150	43.5	.5	13.5	10.4	13.6	10.2	.11	11.0	43	89	17.0	12.1	3.57
							August						
4,850	-9.8	4.6	19.4	16.6	19.4	16.5	0.05	8.8	11	295	25.5	18.1	0.61
9,850	-17.9	6.9	21.8	19.3	21.9	19.2	-.04	171.7	19	291	29.1	20.6	.92
19,000	-28.4	6.9	25.3	23.4	25.3	23.3	.02	8.5	29	284	34.5	24.4	1.19
31,450	-46.2	5.8	38.3	42.6	45.5	34.9	.24	57.0	47	277	57.3	40.5	1.16
34,055	-52.4	8.4	41.4	48.4	51.2	37.9	.25	61.0	53	279	63.7	45.4	1.16
38,735	-56.3	9.8	43.6	52.5	55.3	40.0	.25	64.0	57	280	68.2	48.2	1.18
43,650	-45.7	8.1	39.1	40.3	45.1	33.5	.29	48.1	46	280	56.2	39.7	1.16
52,800	-19.9	6.1	23.1	18.1	23.5	17.6	.15	16.1	21	287	29.3	20.8	1.01
66,670	15.0	-.7	12.5	9.3	12.6	9.2	.11	10.0	15	93	15.6	11.0	1.36
79,650	27.8	-.7	11.9	9.3	11.9	9.3	-.01	178.2	30	91	15.1	10.7	2.81
86,150	36.6	-.5	13.2	10.3	13.2	10.2	-.05	174.4	37	91	16.7	11.8	3.12
							September						
4,850	-10.9	1.4	19.8	17.9	19.9	17.8	0.05	13.6	11	277	26.7	18.9	0.58
9,850	-21.7	.1	22.4	18.8	22.5	18.6	.07	10.6	22	270	29.2	20.7	1.06
19,000	-36.5	-2.5	25.7	23.3	26.6	22.2	.15	28.4	37	266	34.7	24.5	1.51
31,450	-59.3	-10.9	38.0	41.0	45.5	32.5	.32	51.9	60	260	55.9	39.5	1.52
34,055	-69.0	-9.2	43.2	49.3	53.3	38.1	.30	57.0	70	262	65.5	46.4	1.51
38,735	-74.7	-8.4	44.6	51.0	53.2	41.9	.20	62.0	75	264	67.8	47.9	1.57
43,650	-68.4	-8.4	40.0	41.8	45.0	36.3	.21	51.2	69	263	57.9	40.9	1.69
52,800	-38.8	-2.7	24.9	22.0	26.1	20.5	.21	29.3	39	266	33.2	23.5	1.66
66,670	-.6	-2.3	15.0	10.8	15.1	10.7	.11	8.8	2	194	18.5	13.1	.15
79,650	12.0	-1.2	13.8	8.6	13.9	8.4	.17	9.6	12	96	16.3	11.5	1.04
86,150	14.0	-.5	15.6	10.3	15.6	10.3	.03	2.0	14	92	18.7	13.2	1.06

TABLE II.- STATISTICAL PARAMETERS FOR WIND DISTRIBUTIONS

AT THE NASA WALLOPS STATION - Concluded

Altitude, ft	$\bar{u}$, fps	$\bar{v}$, fps	σ_u, fps	σ_v, fps	σ_ξ, fps	σ_η, fps	ρ	ψ, deg	$\bar{V}$, fps	$\bar{\theta}$, deg	σ_V, fps	σ, fps	$\frac{\bar{V}}{\sigma}$
						October							
4,850	-8.7	2.9	21.4	22.1	22.8	20.6	0.10	53.7	9	288	30.8	21.8	0.41
9,850	-17.5	-6.5	24.7	25.5	26.3	24.0	.09	55.2	18	268	35.5	25.1	.72
19,000	-32.0	-6.1	31.8	37.7	39.5	29.5	.23	63.0	33	259	49.3	34.9	.95
31,450	-49.9	-15.5	43.7	56.2	57.5	42.0	.18	72.3	52	253	71.2	50.3	1.03
34,055	-55.5	-13.4	46.2	58.8	59.9	44.8	.16	74.0	74	256	74.8	52.9	1.08
38,735	-61.2	-9.7	43.6	53.7	54.9	42.1	.17	71.0	71	261	69.2	48.9	1.27
43,650	-59.3	-10.4	38.0	46.1	47.8	35.8	.21	66.6	60	260	59.7	42.3	1.42
52,800	-38.5	-4.4	24.4	26.7	28.0	22.9	.18	58.6	39	263	36.2	25.6	1.52
66,670	-14.0	-1.0	16.9	13.3	17.9	11.9	.31	26.0	14	266	21.5	15.2	.92
79,650	-15.3	-2.8	19.2	13.2	20.1	11.7	.36	21.6	16	260	23.3	16.5	.97
86,150	-23.6	-3.3	25.5	12.4	25.6	12.3	.07	2.5	24	262	28.4	20.1	1.20
						November							
4,850	-23.1	-2.4	22.0	25.7	26.1	21.6	0.11	72.3	23	269	33.8	23.9	0.96
9,850	-39.8	-6.8	27.7	30.4	31.8	26.1	.17	59.7	40	260	41.1	29.1	1.38
19,000	-65.4	-13.0	41.8	46.5	48.7	39.2	.19	59.8	67	259	62.5	44.2	1.52
31,450	-95.9	-15.7	57.5	64.7	68.5	52.9	.23	58.7	97	261	86.6	61.2	1.58
34,055	-96.9	-13.8	58.0	66.0	68.9	54.5	.20	62.0	98	262	87.9	62.1	1.58
38,735	-95.2	-11.5	53.1	60.9	61.9	51.9	.11	71.0	96	263	80.8	57.1	1.68
43,650	-92.3	-10.2	46.9	47.2	50.4	43.4	.15	46.0	93	264	66.5	47.1	1.98
52,800	-62.2	-7.1	31.8	31.5	33.8	29.3	.14	43.0	63	263	44.8	31.7	1.99
66,670	-27.3	-3.8	24.2	16.5	24.8	15.6	.24	15.9	28	262	29.3	20.7	1.35
79,650	-27.3	-.1	31.1	17.2	32.0	15.4	.39	15.8	27	270	35.5	25.1	1.07
86,150	-38.9	4.8	31.4	19.9	32.6	18.0	.35	18.3	39	270	37.2	26.3	1.48
						December							
4,850	-30.1	-0.3	20.5	27.9	28.3	20.0	0.15	76.8	30	269	34.6	24.5	1.23
9,850	-50.6	-3.2	22.7	32.4	32.5	22.6	.06	85.5	51	266	39.6	28.0	1.82
19,000	-82.3	-7.7	35.1	51.1	51.1	35.0	.06	85.8	83	265	62.0	43.8	1.89
31,450	-120.4	-8.9	53.1	67.9	69.0	51.5	.16	73.9	121	266	86.2	61.0	1.98
34,055	-128.4	-6.1	58.6	67.2	68.9	56.6	.14	68.0	129	267	89.2	63.1	2.05
38,735	-122.2	-2.0	53.5	55.8	58.8	50.3	.15	53.0	122	269	77.3	54.7	2.23
43,650	-117.4	-2.9	47.0	45.8	49.5	43.1	.13	39.4	117	269	65.6	46.4	2.52
52,800	-80.7	-3.3	35.3	31.4	36.3	30.2	.14	25.2	81	268	47.2	33.4	2.42
66,670	-34.3	.8	31.8	17.7	32.0	17.3	.18	8.0	34	271	36.3	25.7	1.32
79,650	-29.4	1.7	41.4	19.0	41.5	18.8	.15	4.9	29	273	45.6	32.2	.90
86,150	-31.1	.3	51.4	21.2	52.3	18.9	.42	11.5	31	271	55.6	39.3	.79

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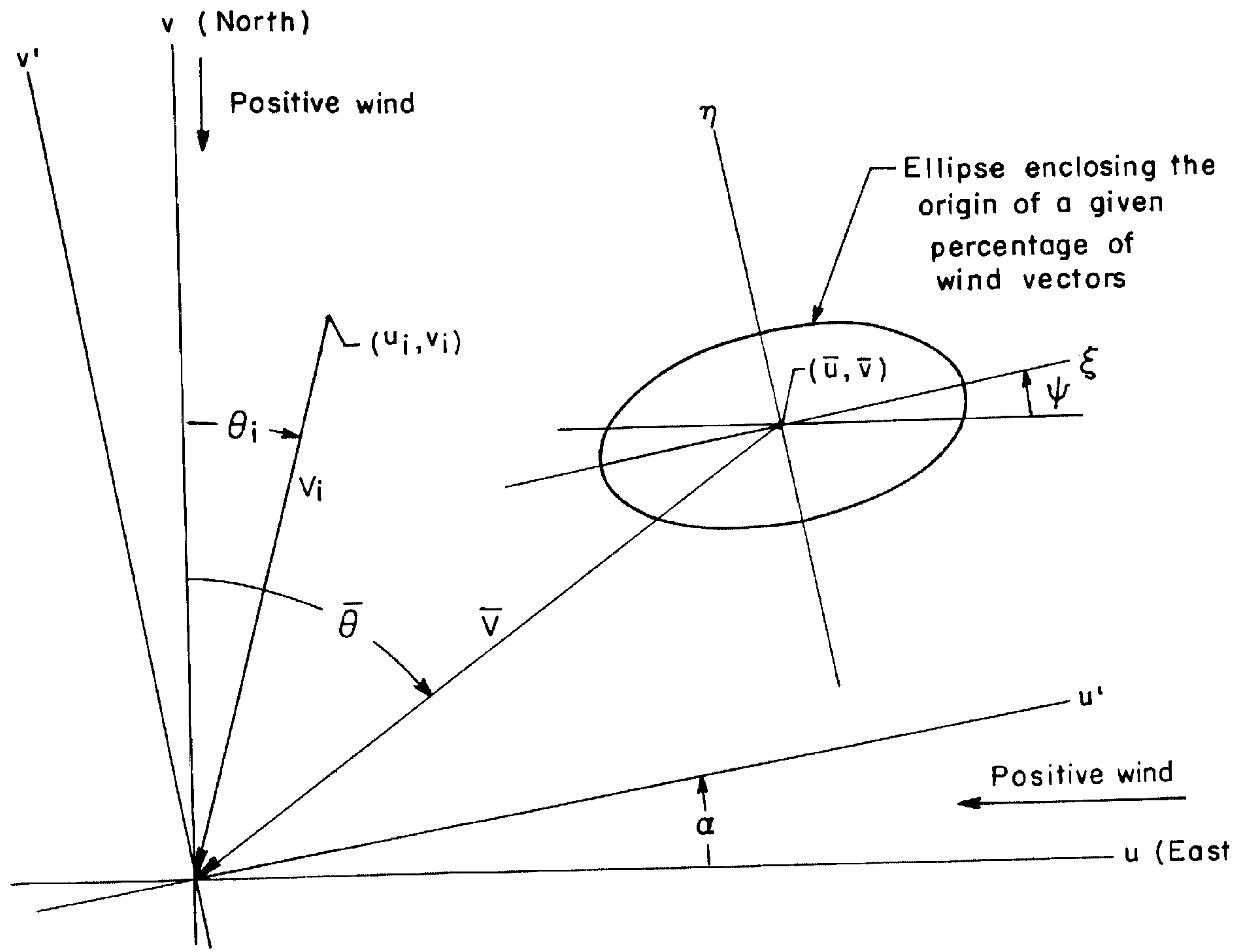
TABLE III.- TYPICAL VALUES OF STATISTICAL PARAMETERS

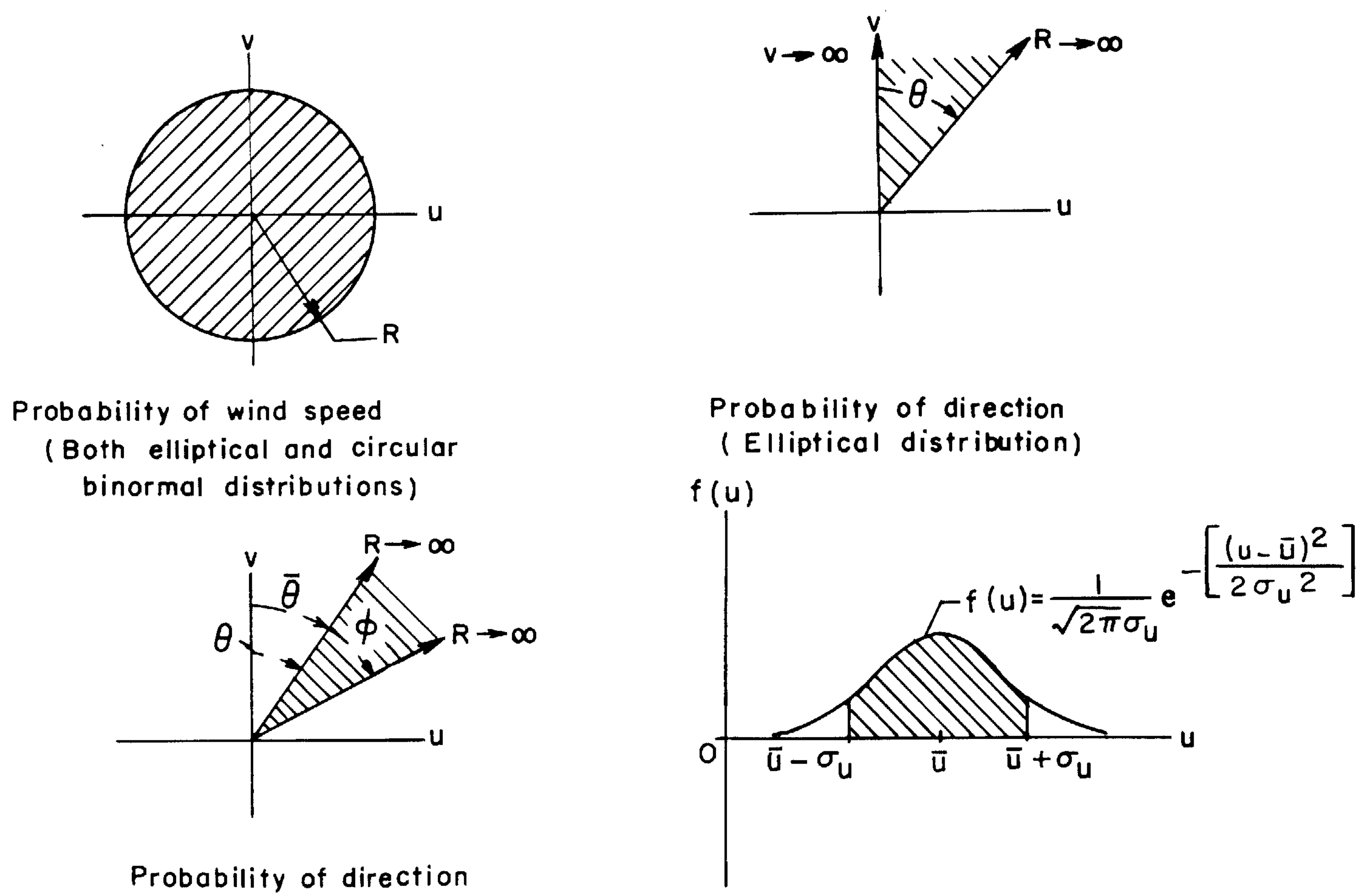
FOR WASHINGTON, D.C., AND NORFOLK, VA.

Quantity	Value of statistical parameter for -	
	Norfolk, Va.	Washington, D.C.
January; 31,450-foot altitude		
N	616	589
$\sum$ u, fps	-81,664.4	-67,650.6
$\sum$ v, fps	3,260.3	3,070.9
$\sum$ uv, $(\text{fps})^2$	-371,140.0	-510,483.2
$\sum (u^2)$, $(\text{fps})^2$	13,228,746.5	10,464,404.8
$\sum (v^2)$, $(\text{fps})^2$	2,501,329.8	2,839,305.9
July; 19,000-foot altitude		
N	584	710
$\sum$ u, fps	-14,742.2	-21,669.8
$\sum$ v, fps	4,237.8	7,138.2
$\sum$ uv, $(\text{fps})^2$	-62,040.1	-173,577.5
$\sum (u^2)$, $(\text{fps})^2$	745,646.9	1,122,819.0
$\sum (v^2)$, $(\text{fps})^2$	259,001.7	381,860.9



(a) Coordinate system.

Figure 1.- Geometric description of wind-distribution parameters.



Probability of wind speed (Both elliptical and circular binormal distributions)

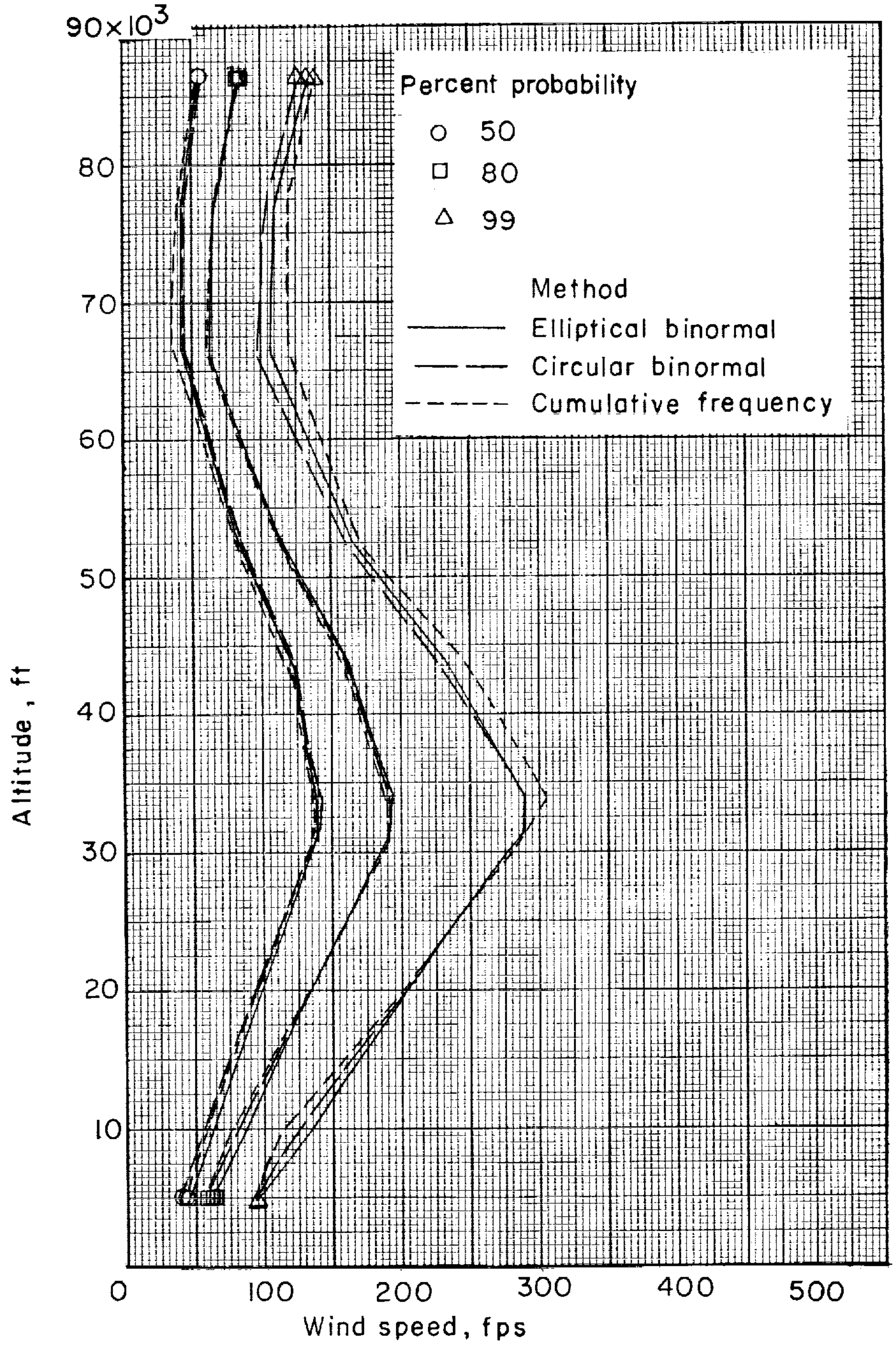
Probability of direction (Elliptical distribution)

Probability of direction (Circular distribution)

Component winds along u-axis (Marginal distribution)

(b) Areas of integration of probability distributions.

Figure 1.- Concluded.



(a) January; 50, 80, 99 percent.

Figure 2.- Comparison of probability wind-speed profiles for Wallops Island obtained by three methods.

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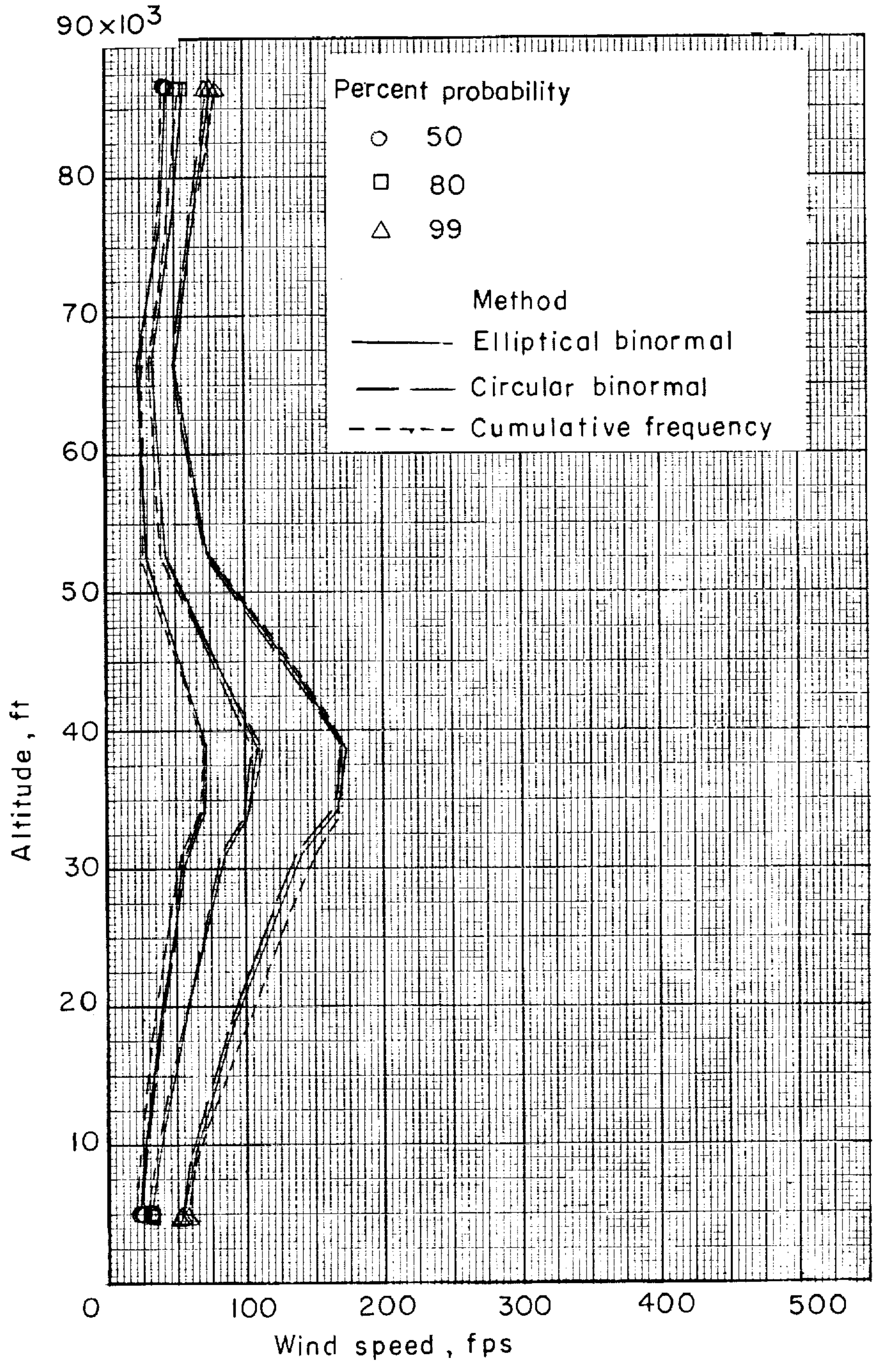
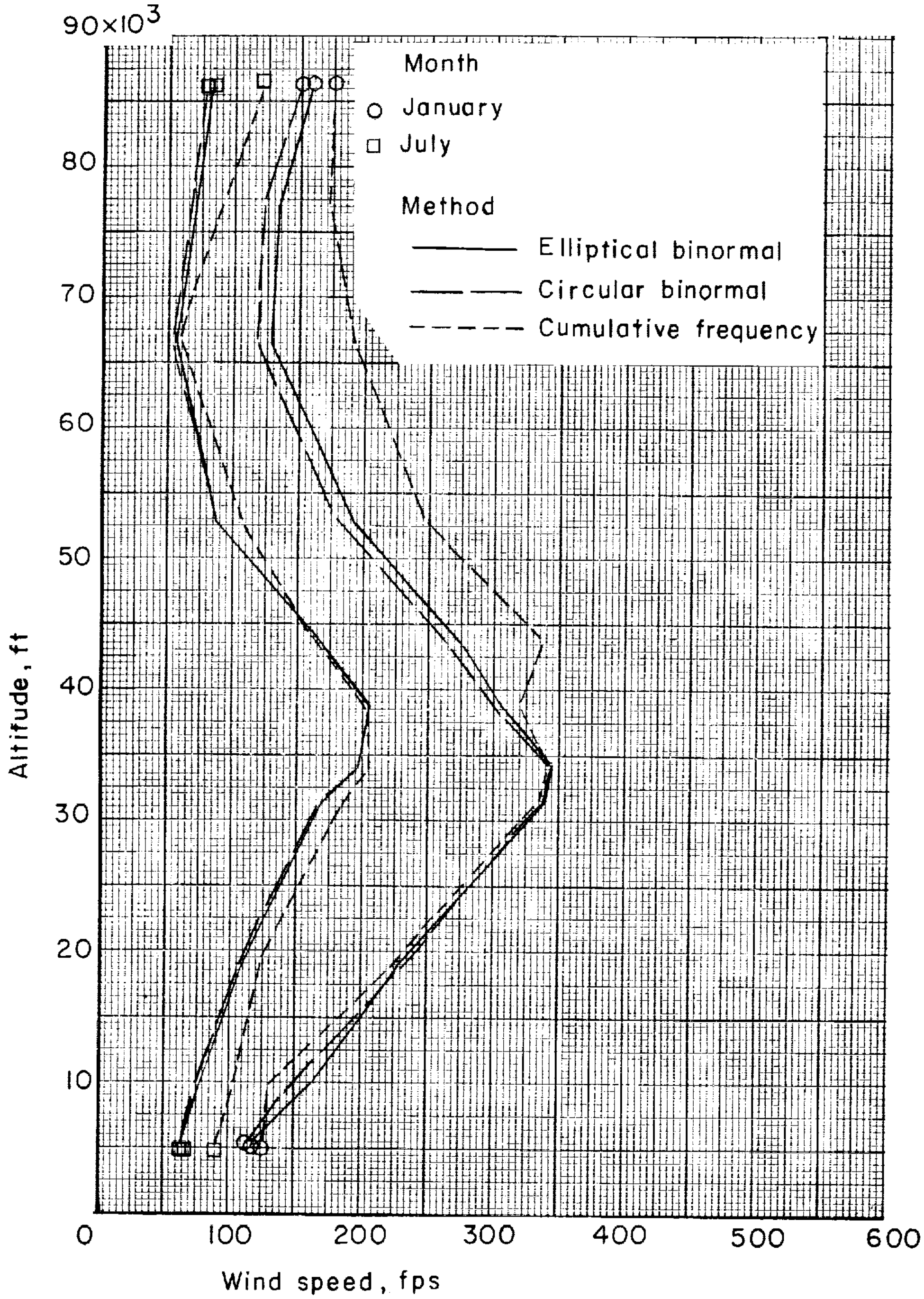


(b) July; 50, 80, 99 percent.

Figure 2.- Continued.



(c) January and July; 99.9 percent.

Figure 2.- Concluded.

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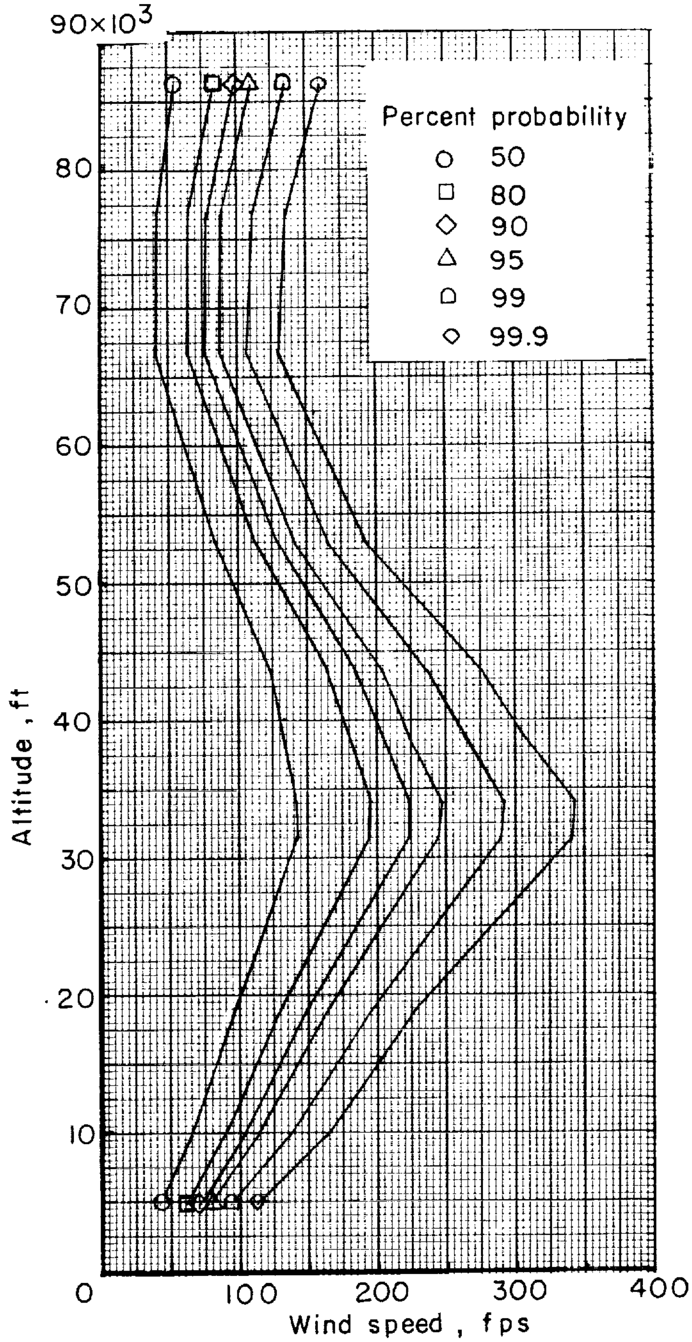
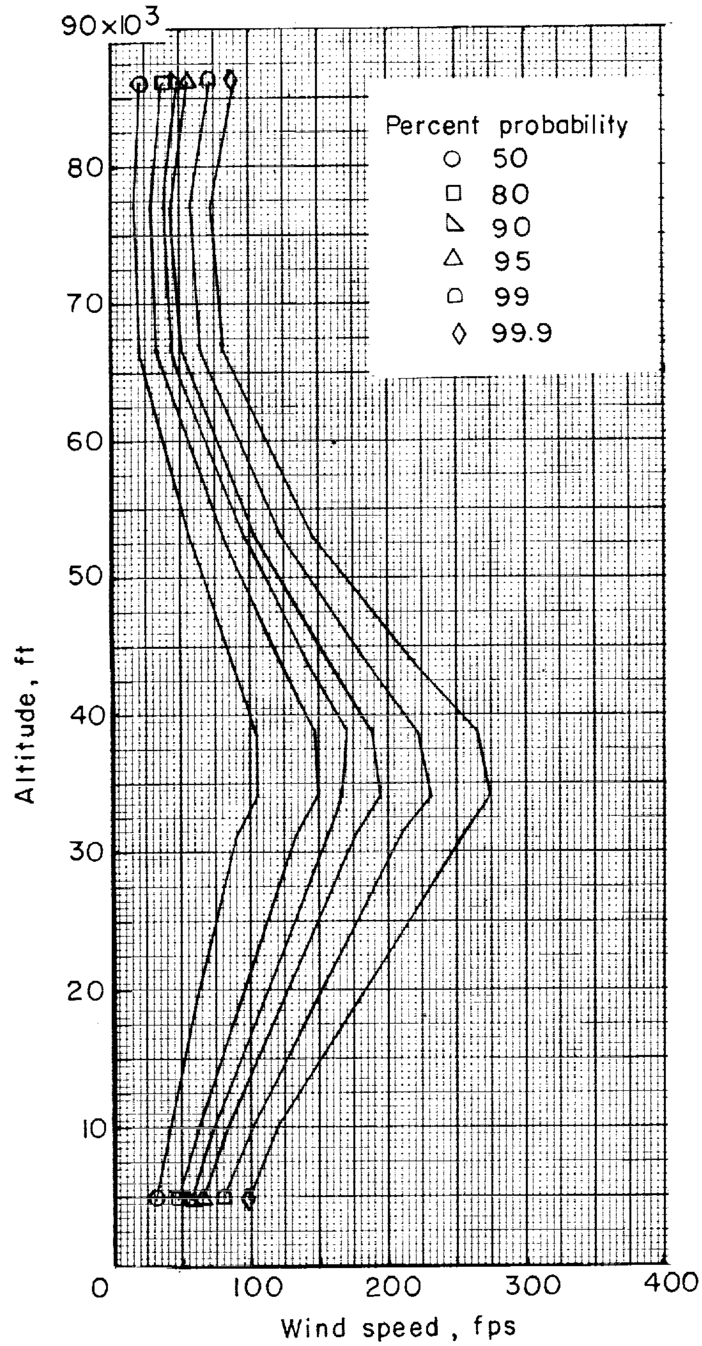


(a) January.

Figure 3.- Probability wind-speed profiles for Wallops Island obtained by the elliptical binormal method.



(b) April.

Figure 3.- Continued.

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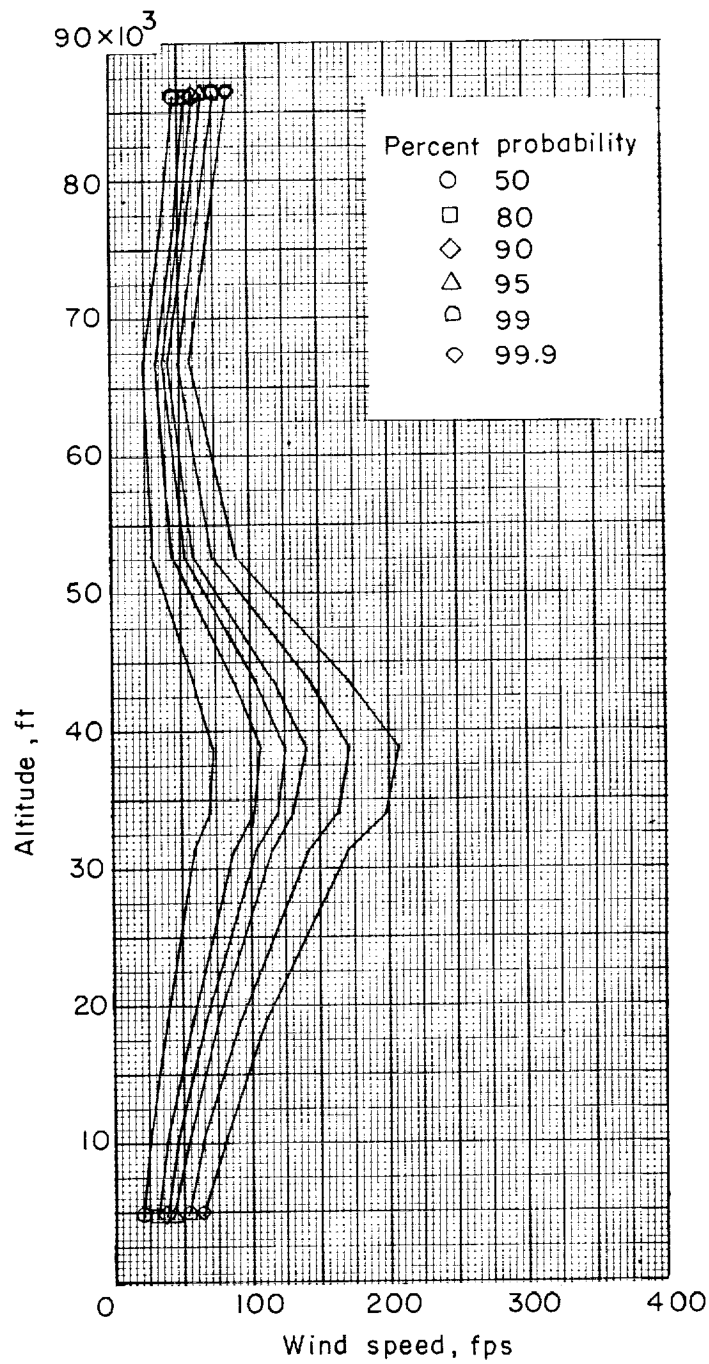
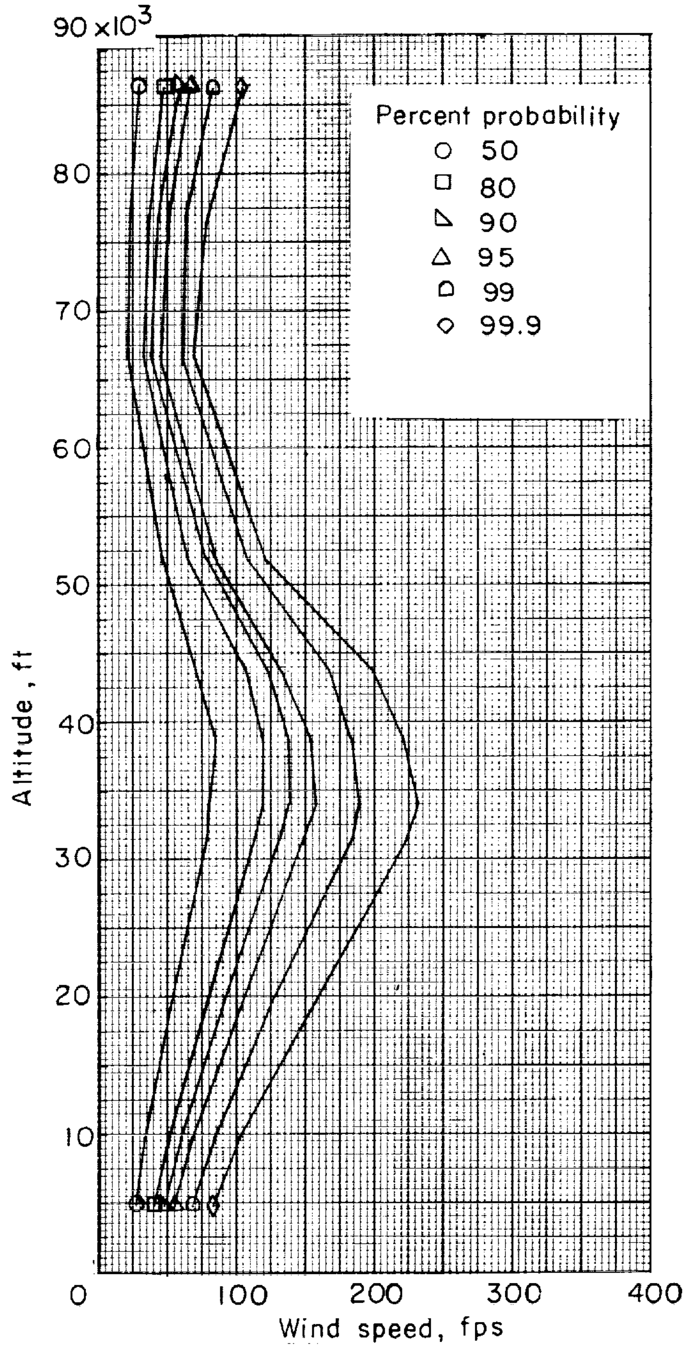


(c) July.

Figure 3.- Continued.



(d) October.

Figure 3.- Concluded.

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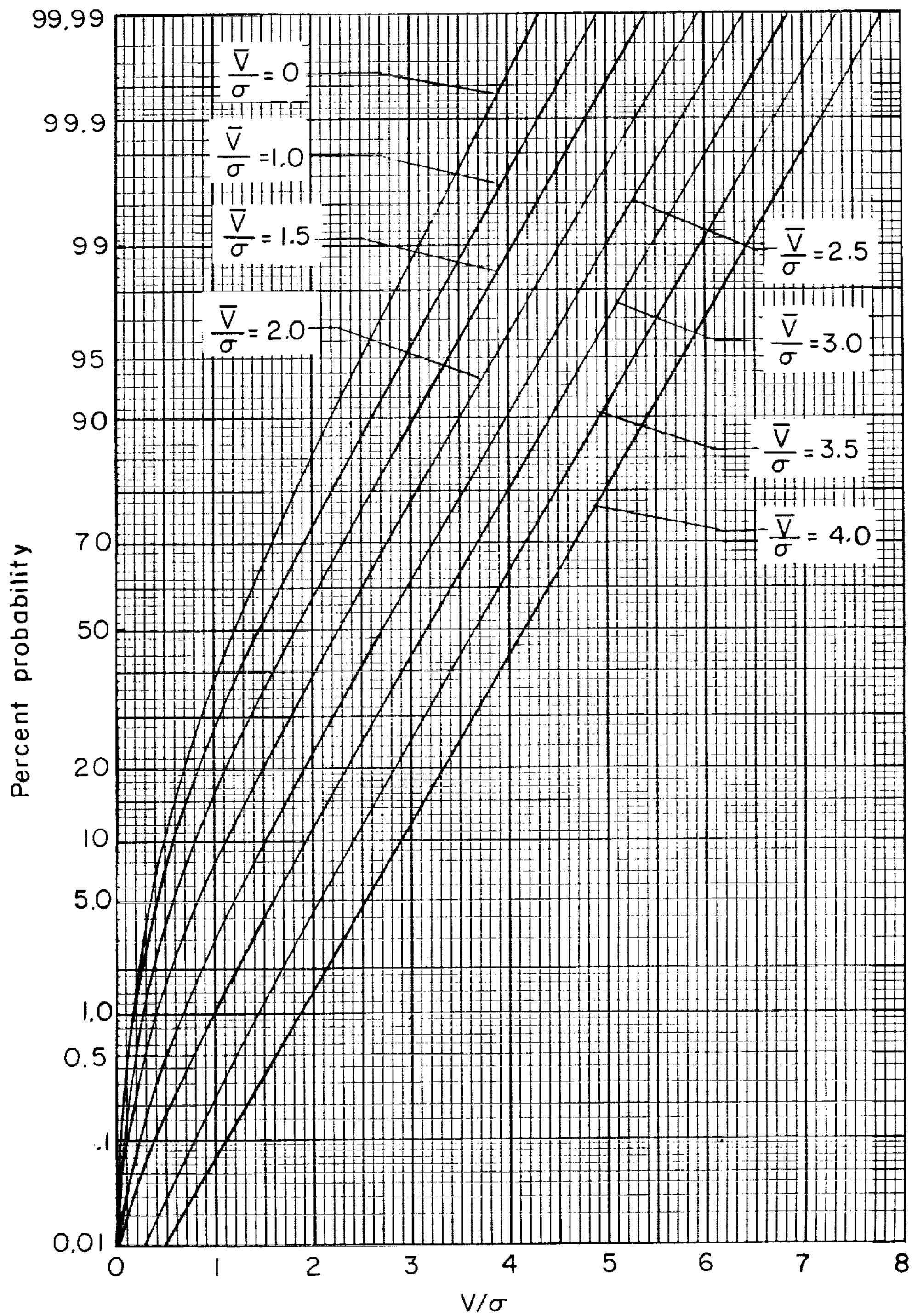
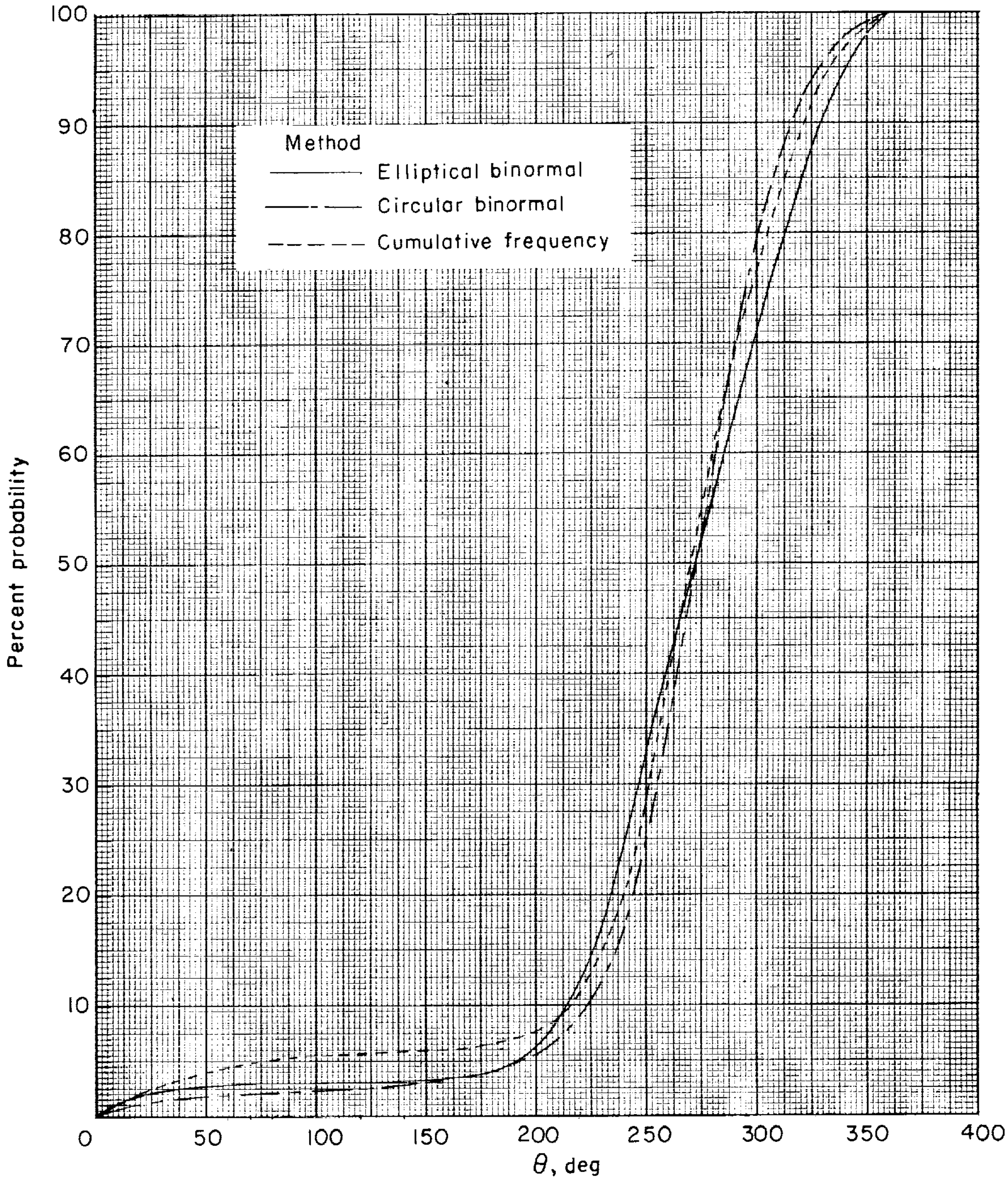


Figure 4.- Wind-speed probability curves for circular binormal distributions.



(a) January; 9,850-foot altitude.

Figure 5.- Comparison of wind-direction probability curves for Wallops Island obtained by three methods.

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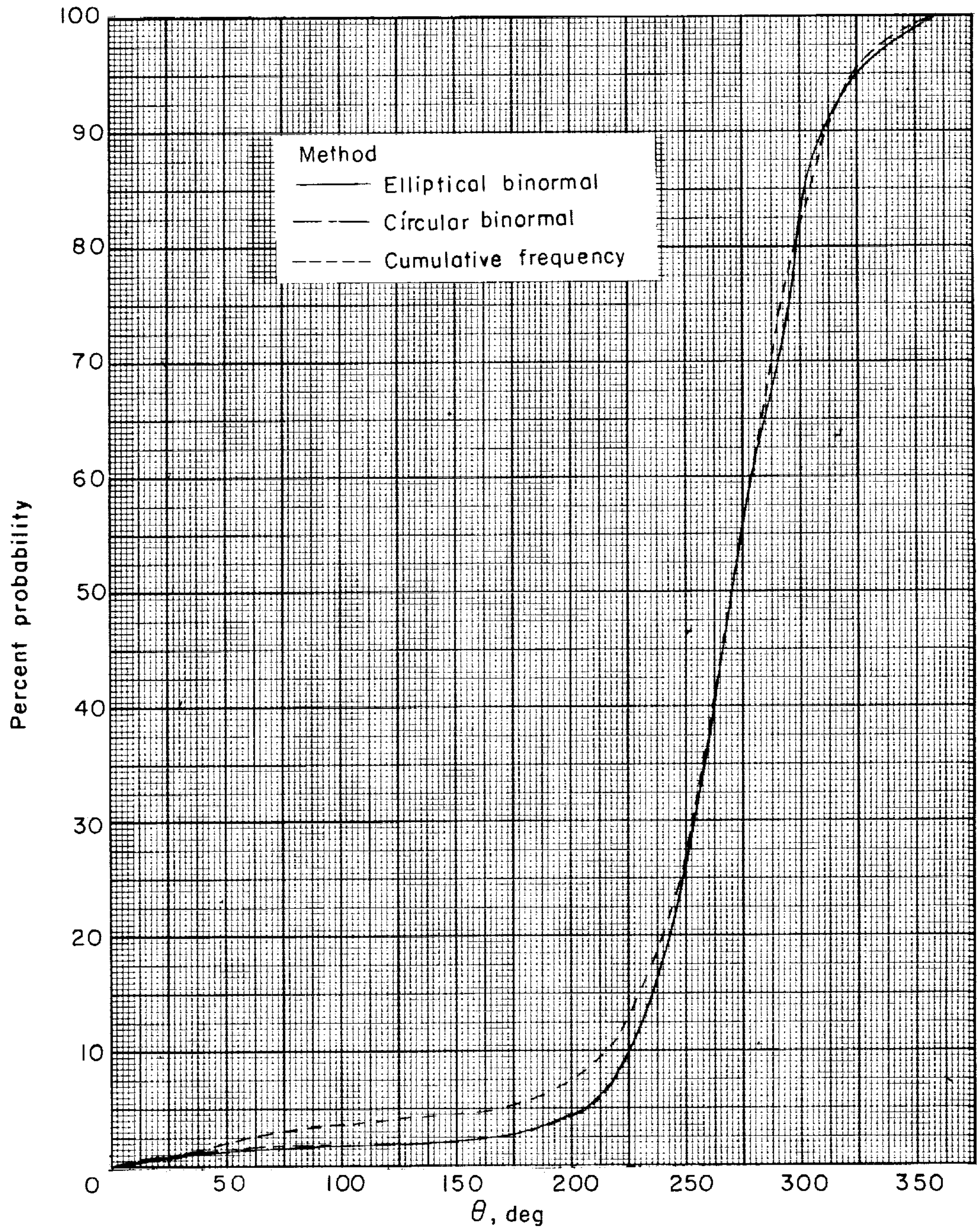
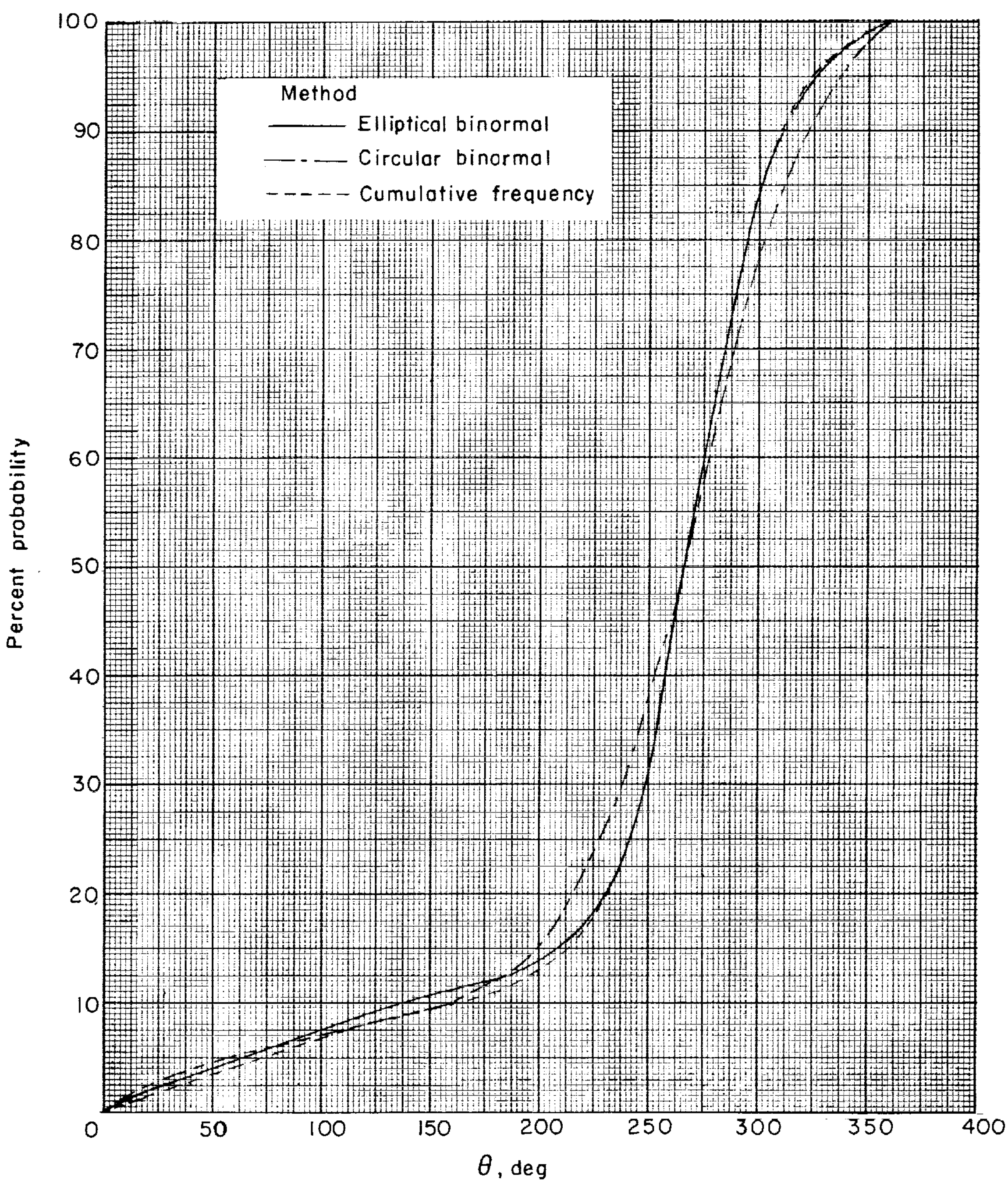


(b) January; 31,450-foot altitude.

Figure 5.- Continued.



(c) January; 66,670-foot altitude.

Figure 5.- Concluded.

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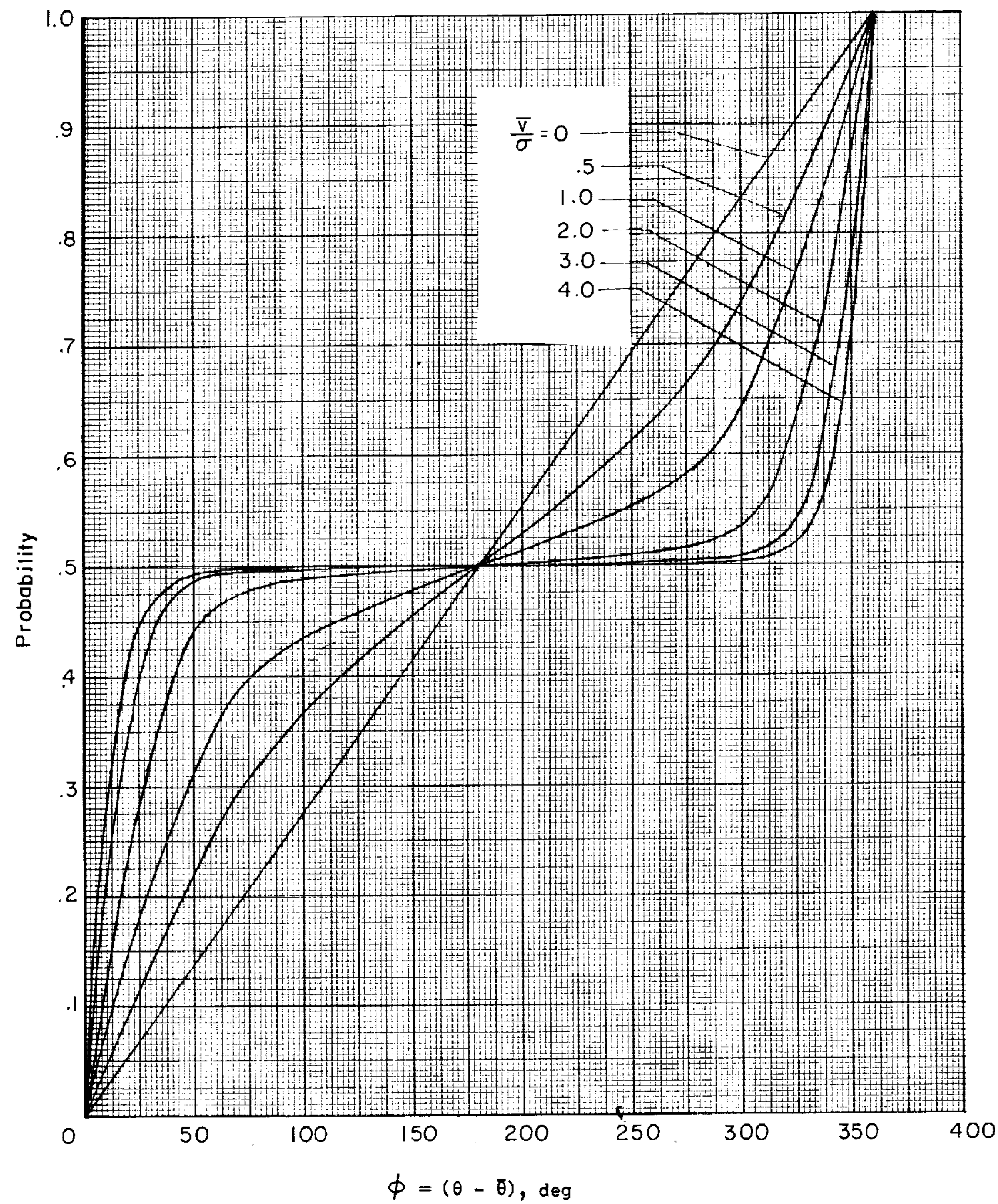
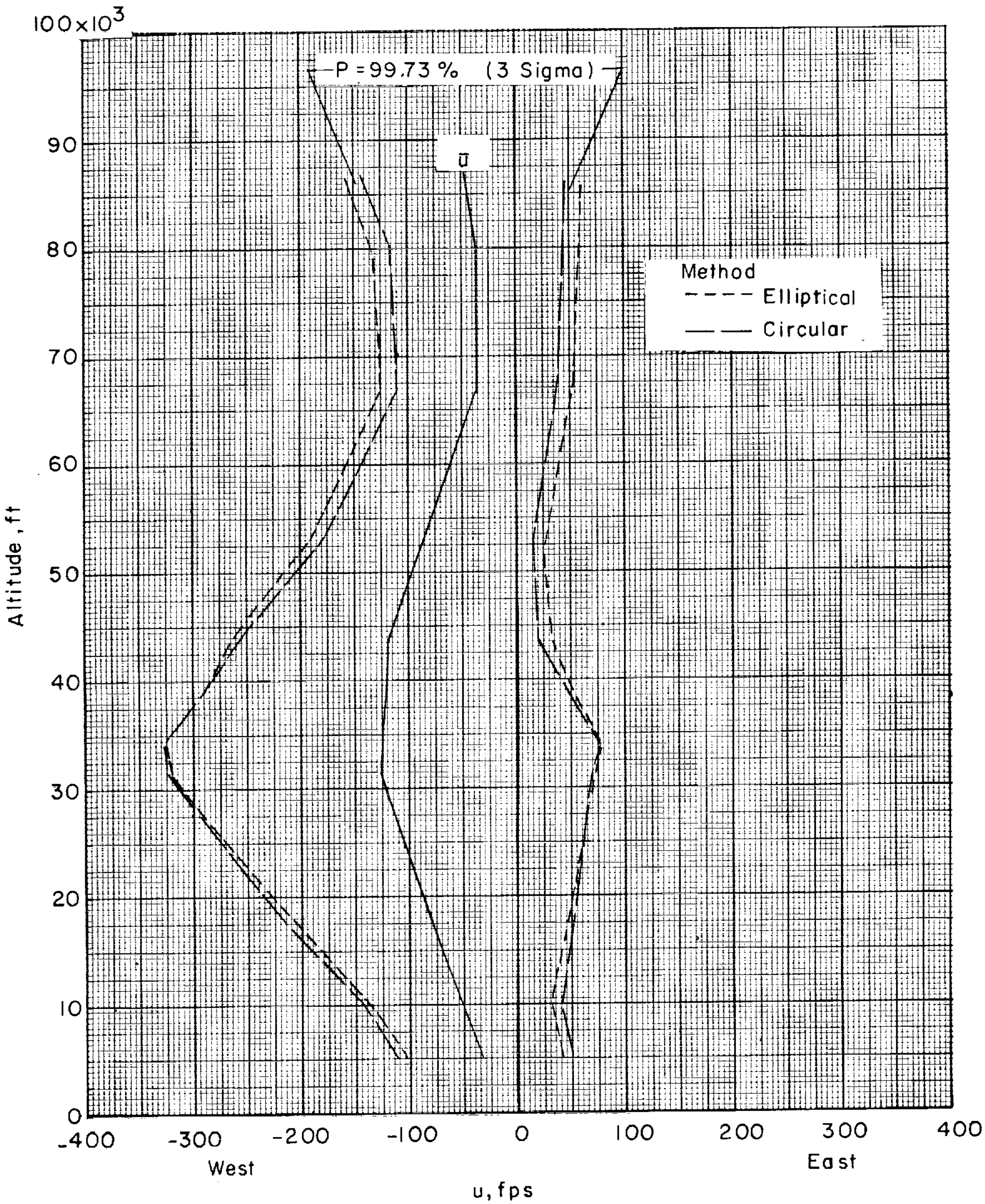


Figure 6.- Wind-direction probability curves for circular binormal distributions.



(a) January east-west winds.

Figure 7.- Comparison of component wind envelopes for Wallops Island.

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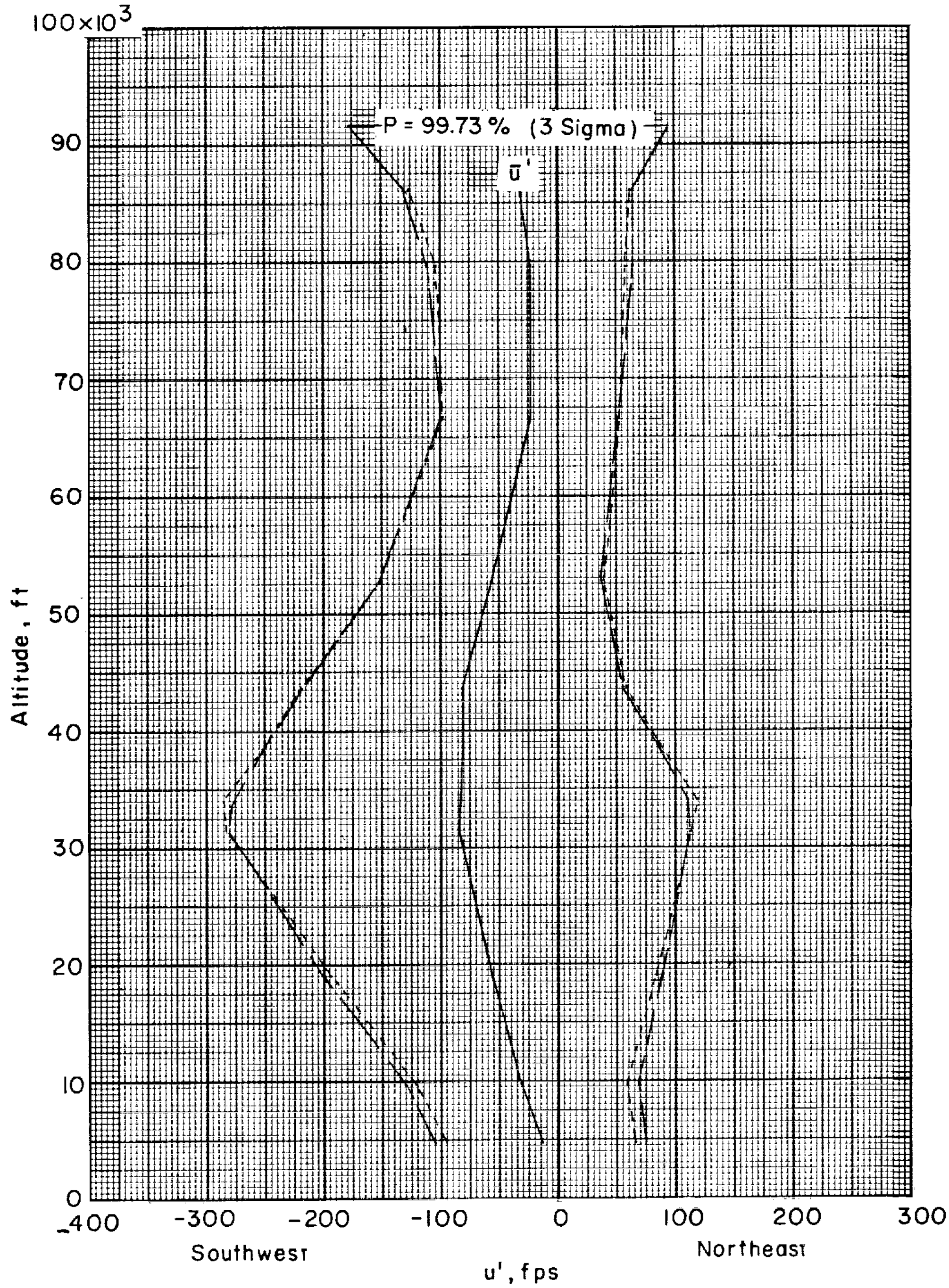
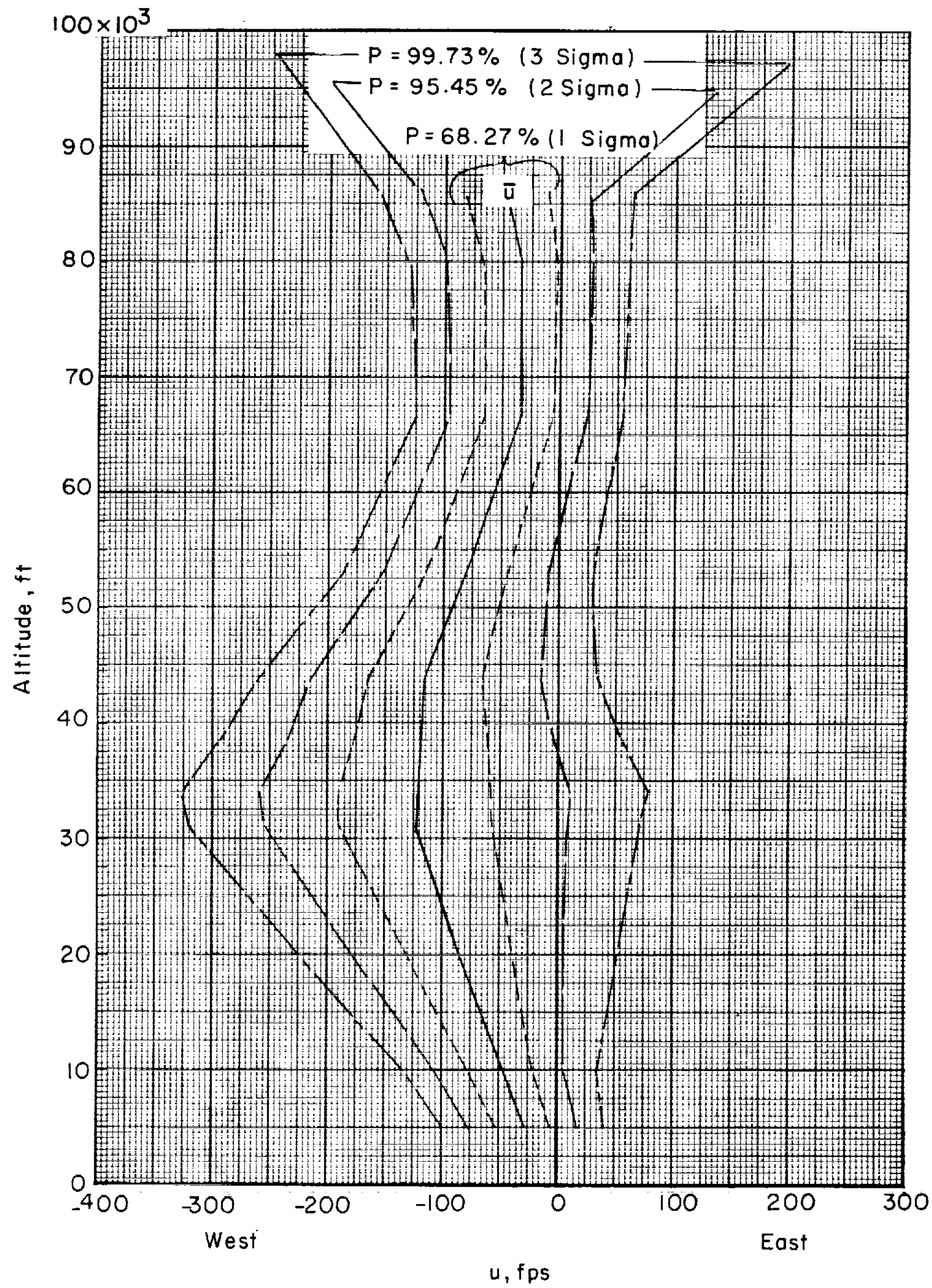


(b) January northeast-southwest winds.

Figure 7.- Concluded.



(a) January east-west wind components.

Figure 8.- Probability envelopes for component winds at Wallops Island. (Elliptical distribution method.)

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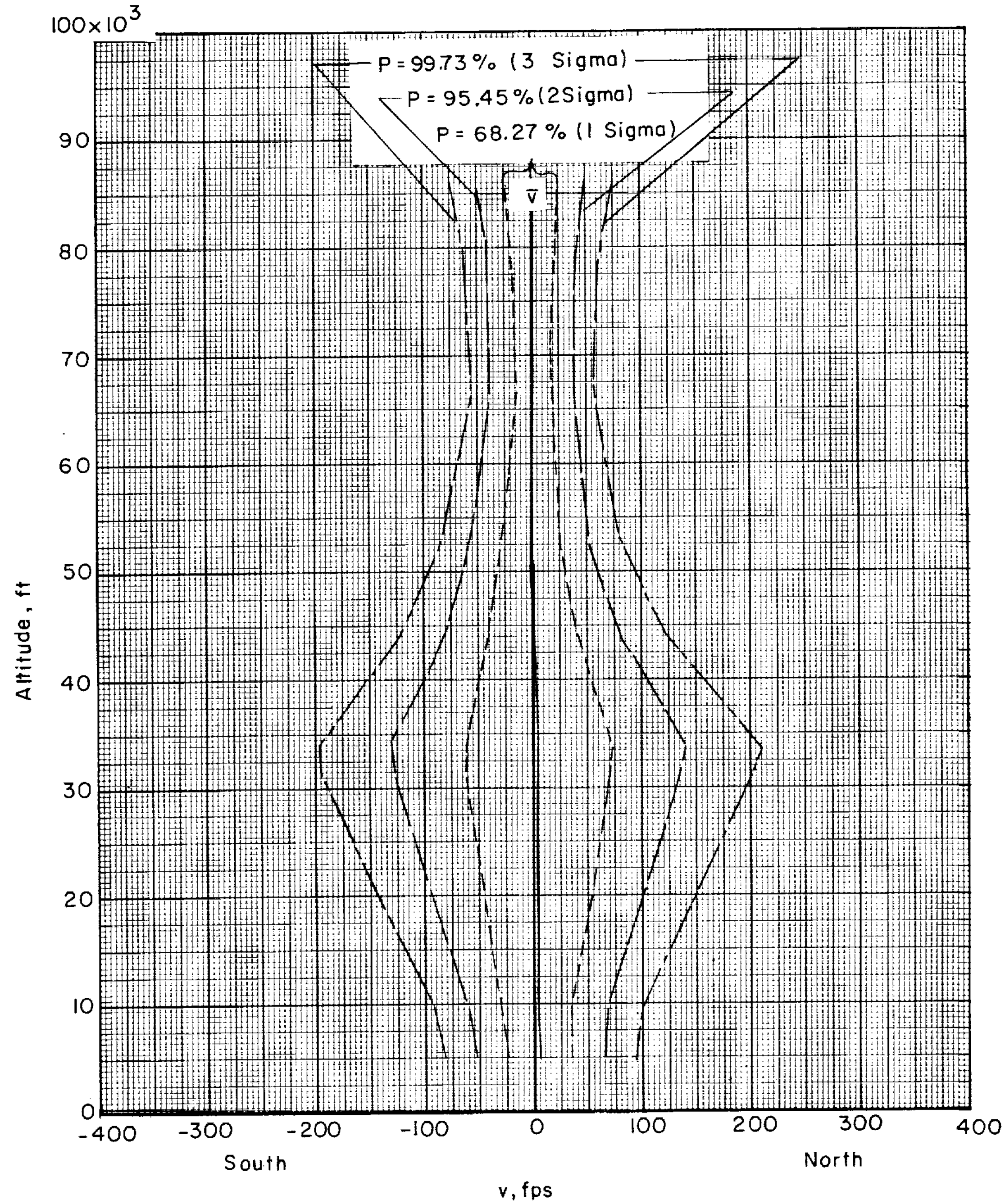
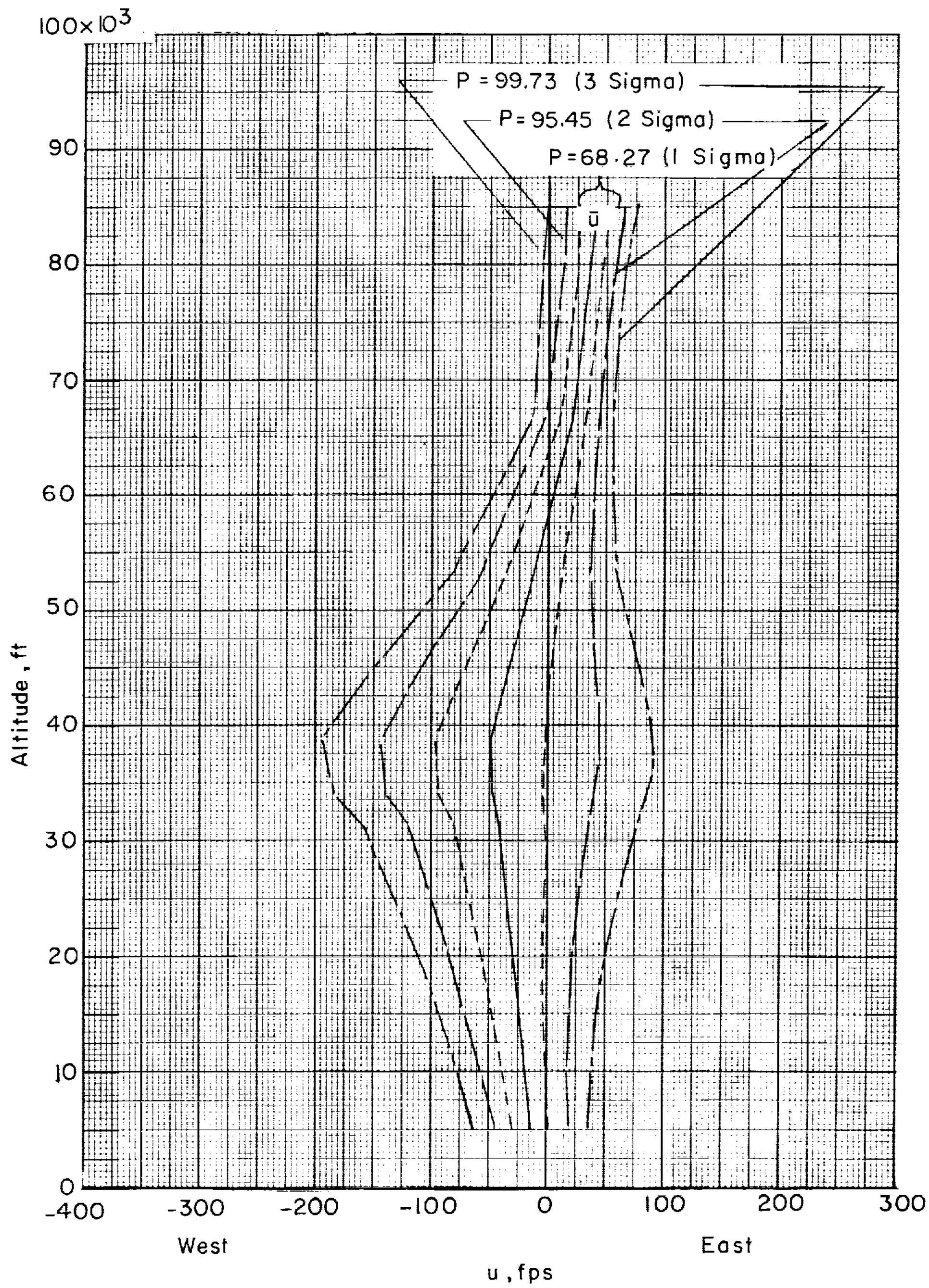


(b) January north-south wind components.

Figure 8.- Continued.



(c) July east-west wind components.

Figure 8.- Continued.

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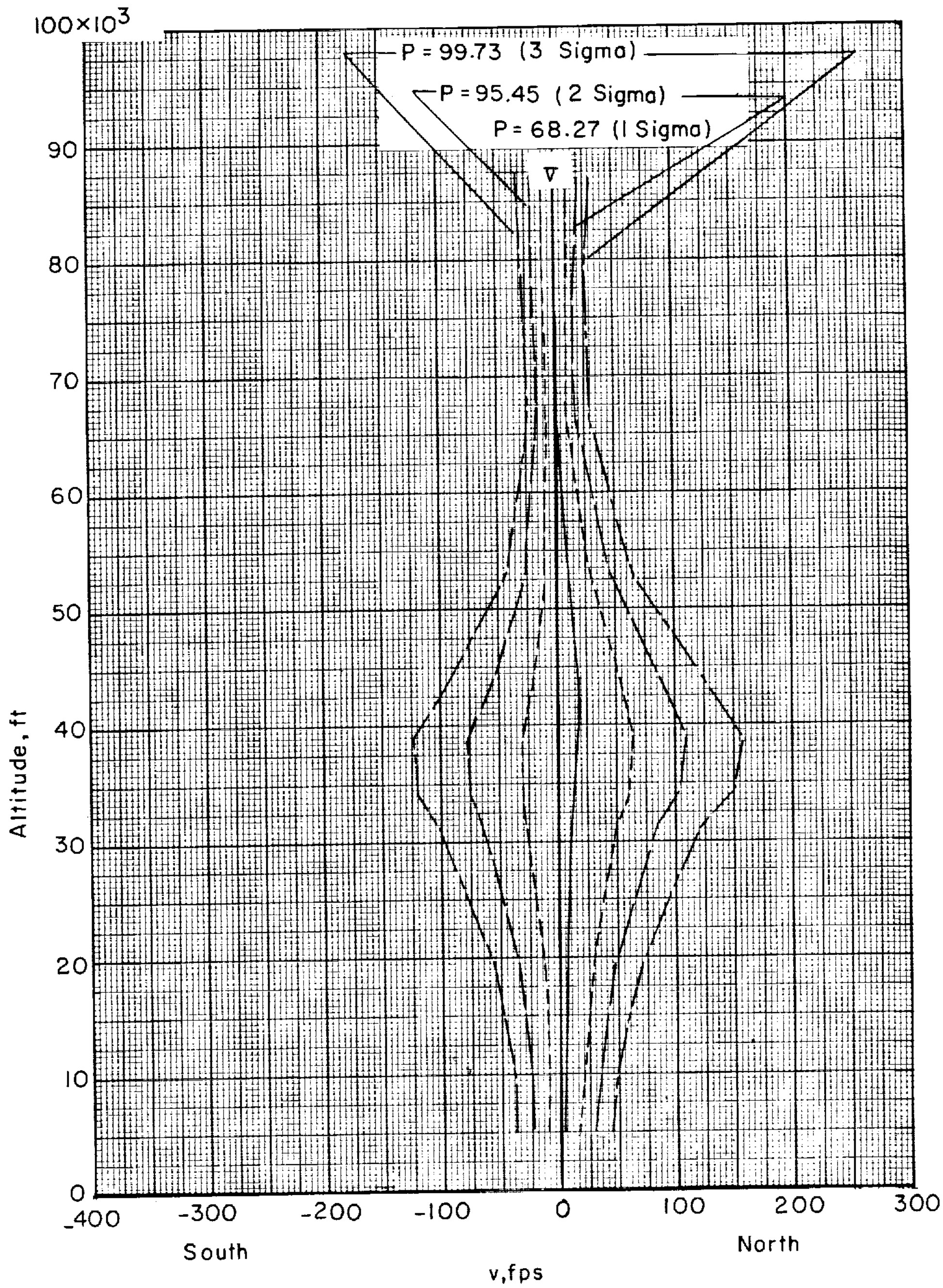
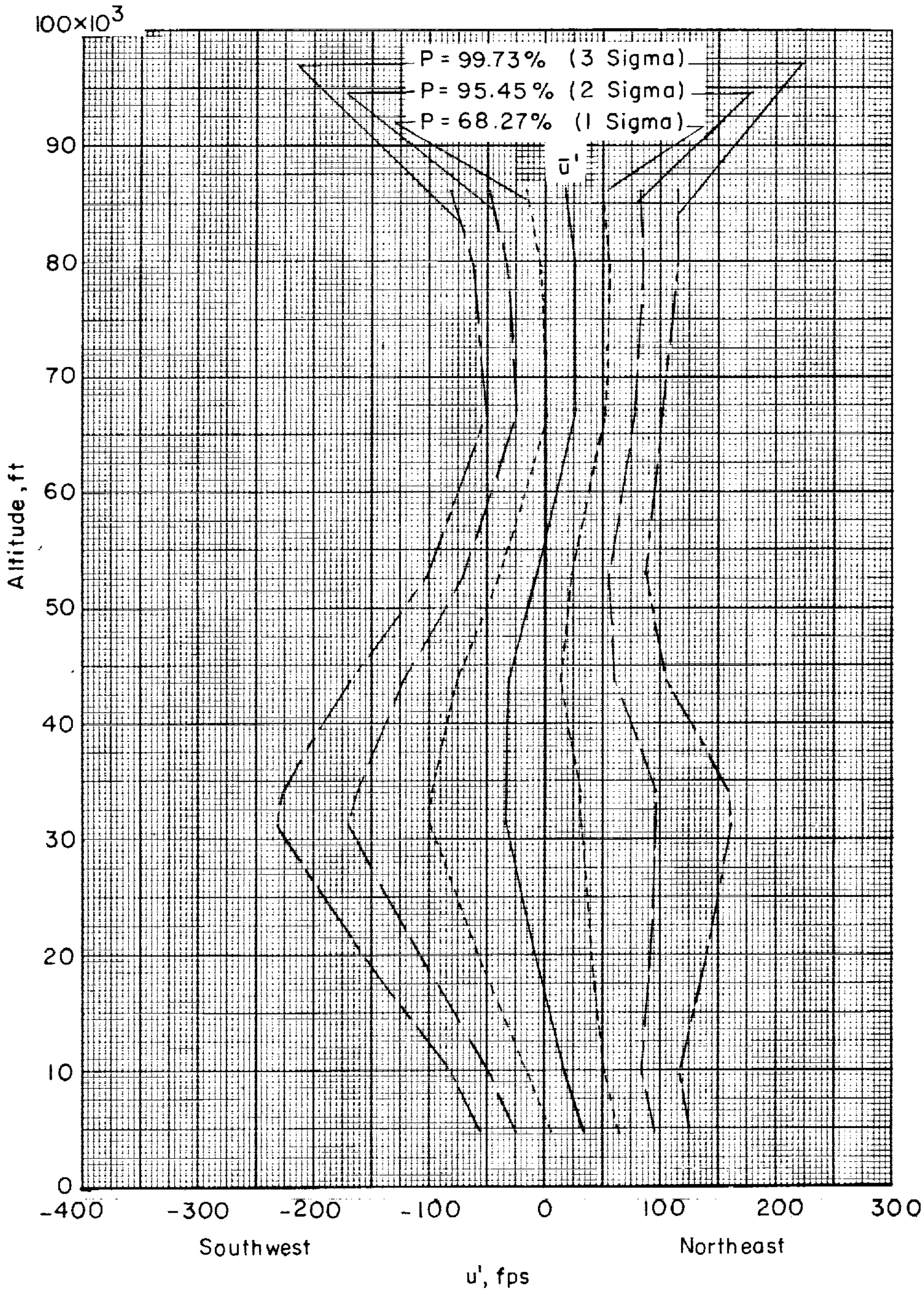


(d) July north-south wind components.

Figure 8.- Continued.



(e) January northeast-southwest wind components.

Figure 8.- Continued.

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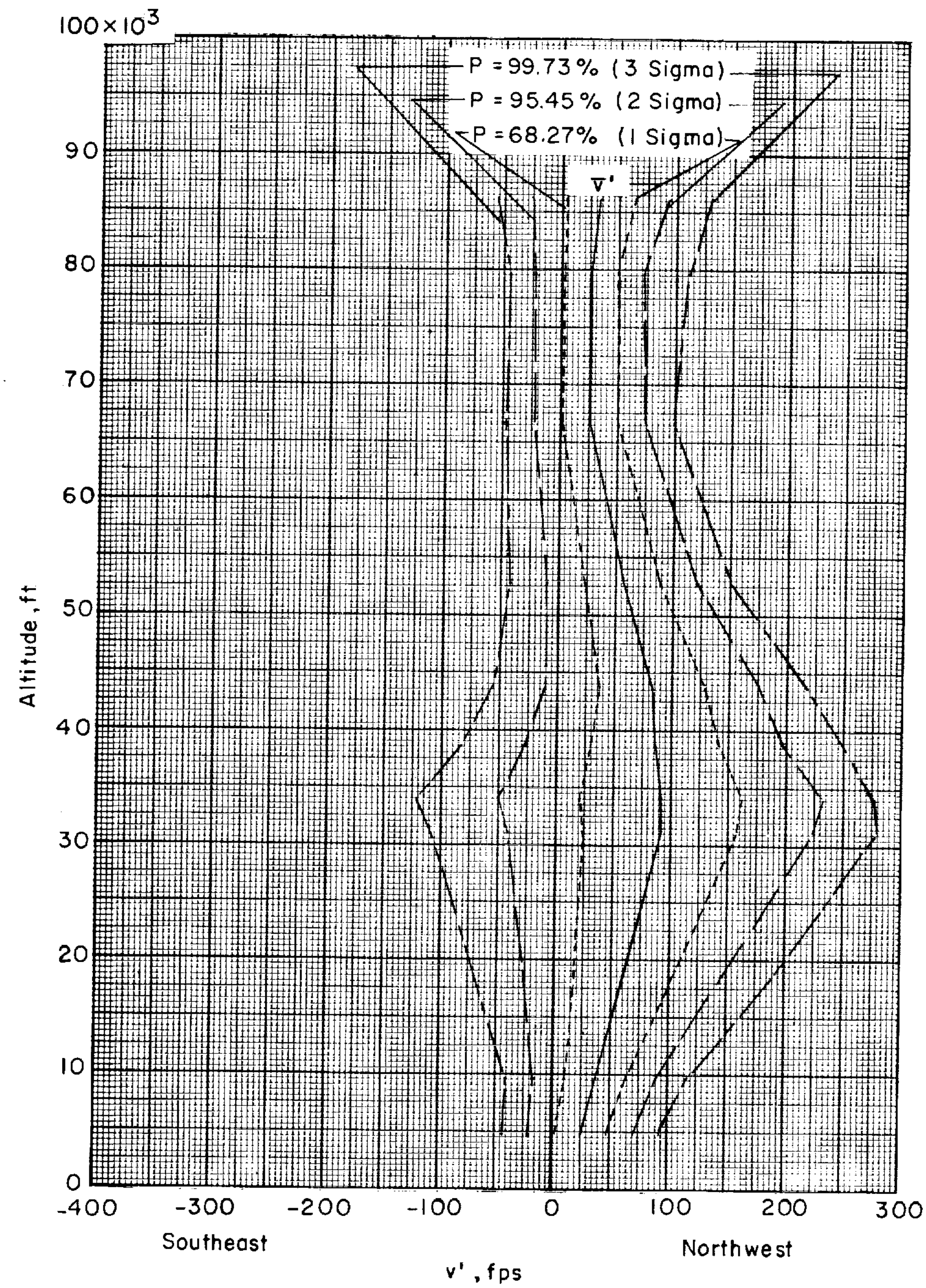

(f) January northwest-southeast wind components.

Figure 8.- Concluded.

NASA TN D-1249
National Aeronautics and Space Administration.
STATISTICAL WIND DISTRIBUTION DATA FOR USE AT NASA WALLOPS STATION. William L. Weaver, Andrew G. Swanson, and John F. Spurling. July 1962. 55p. OTS price, $1.50.
(NASA TECHNICAL NOTE D-1249)

Wind data from radiosonde measurements made at Norfolk, Virginia, and Washington, D.C., were utilized to determine statistical wind distribution data for an altitude range from 4,850 to 86,150 feet. Some of the results obtained by a binormal analysis are compared with results obtained empirically. The assumption of a circular distribution is shown to be valid for Wallops Island winds. A description of the method of interpolating the original data from Washington and Norfolk and an outline of the statistical methods employed are included. Simple methods are discussed for obtaining circular distribution wind speed, wind direction, and component wind probabilities and also elliptical distribution component probabilities.

I. Weaver, William L.
II. Swanson, Andrew G.
III. Spurling, John F.
IV. NASA TN D-1249

(Initial NASA distribution: 21, Geophysics and geodesy; 23, Launching facilities and operations; 45, Research and development facilities.)

NASA

NASA TN D-1249
National Aeronautics and Space Administration.
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